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★**Lectures on curves, surfaces and projective varieties.**
A classical view of algebraic geometry.
Translated from the 2003 Italian original by Francis Sullivan.
EMS Textbooks in Mathematics.

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The book under review differs substantially from the recent avalanche of introductory textbooks in algebraic geometry. It is a very nice introduction to the subject of the classical methods in algebraic geometry, explained in a modern way. It is not a surprise these days when a student feels at ease with modern theories such as stacks or derived categories but has little knowledge about numerous beautiful concrete facts and constructions in algebraic geometry which have accumulated over almost two centuries. The reviewer cannot agree more with the following statement of the authors:

"The topics chosen throw light on intuitive concepts that were a starting point for much contemporary research, and should therefore, in our opinion, make up a part of much of the cultural baggage of any young student intending to work in algebraic geometry. Our hope is that this text, which can be a first step in recovering an important and fascinating patrimony of mathematical ideas, will stimulate in some readers the desire to look into the original work of great geometers of the past, and perhaps even to find therein motivation for significant new research."

The book differs substantially from the reviewer's forthcoming treatise on classical algebraic geometry (available on the web) which represents more advanced theories and relies on a sophisticated background in modern algebraic geometry. The prerequisites for the present book are very modest: the foundational aspects of projective geometry and basic algebraic structures. Among comparable sources of learning classical algebraic geometry one could refer to J. D. Harris's book [*Algebraic geometry*, Corrected reprint of the 1992 original, Springer, New York, 1995; MR1416564 (97e:14001)] and J. G. Semple and L. Roth's book [*Introduction to algebraic geometry*, Reprint of the 1949 original, Oxford Univ.

Press, New York, 1985; MR0814690 (86m:14001)], although the latter is written in a rather archaic way.

Here is a short description of the contents of the book.

Chapter 1, "Prerequisites", gives a rapid review of the facts from projective geometry which are used throughout the book.

Chapter 2, "Algebraic sets, morphisms, and rational sets", introduces some basic concepts of algebraic geometry: affine and projective algebraic sets, regular and rational maps between them.

Chapter 3, "Geometric properties of algebraic varieties", discusses some fundamental aspects of the theory of algebraic varieties such as dimension and singularities.

Chapter 4, "Rudiments of elimination theory", is aimed at defining the intersection multiplicities of plane algebraic curves and higher-dimensional varieties.

Chapter 5, "Hypersurfaces in projective space", gives, as far as I know, the first introduction to the classical theory of polarity widely practiced by classical geometers and almost forgotten these days.

Chapter 6, "Linear systems", discusses another important tool in classical algebraic geometry in the special but most important case of a linear system of hypersurfaces in a projective space. The approach is down-to-earth but contains many useful facts from the theory, for example, Bertini's first theorem.

Chapter 7, "Algebraic curves", and Chapter 8, "Linear systems on algebraic curves", are the first chapters in which readers start to see some beautiful applications of the previously learnt general facts about algebraic varieties. Although the literature on algebraic curves, both classical and modern, is abundant, readers will be able to get a different view on many standard facts from the theory of algebraic curves.

Chapter 9, "Cremona transformations", gives a short introduction to the theory of birational transformations of projective planes and 3-spaces. This fills a gap in the modern literature on the subject since the existing modern exposition of the theory of plane Cremona transformation by M. Alberich-Carramiñana [*Geometry of the plane Cremona maps*, Lecture Notes in Math., 1769, Springer, Berlin, 2002; MR1874328 (2002m:14008)] is more advanced and technical.

Chapter 10, "Rational surfaces", discusses algebraic surfaces which can be rationally parameterized. Among them are such familiar surfaces as Veronese surfaces, scrolls and del Pezzo surfaces.

Chapter 11, "Segre varieties", is rather short and discusses the geometry of Segre varieties, the embedded products of projective spaces.

Chapter 12, "Grassmann varieties", is devoted to the geometry of lines, mostly in a 3-space. Among the objects discussed in this chapter are ruled surfaces, complexes and congruences of lines.

The final Chapter 13, "Supplementary exercises", is meant to supplement the theories discussed in previous chapters with exercises containing many beautiful concrete examples and facts from classical algebraic geometry. Such exercises also can be also found at the end of Chapters 7–12 and again represent a very nice feature of the book. *I. Dolgachev* (1-MI; Ann Arbor, MI)