## TRADURRE IN ITALIANO IL TESTO SEGUENTE:

## Non-Euclidean geometries

One way to think of hyperbolic geometry is as the geometry that you would see if you lived on a hyperbolic surface. Without being mathematically exact, think about the surface of a saddle for a horse: it slopes up in front and behind you and down where your legs go. The surface of a saddle is rougly the same as a hyperbolic surface.

Now imagine an ant on the saddle which is walking between two points on the surface by the shortest possible route. We humans can see that the ant is moving along a curved path rather than in a perfectly straight Euclidean line. That path will be called a "straight line" in hyperbolic geometry. This makes sense, since one way to think of a straight line is as the shortest distance between two points.

Now imagine three points on the saddle A, B and C and imagine that the ant walks in hyperbolic straight lines from A to B, then from B to C and finally, from C back to A. This path will trace out a "triangle" in hyperbolic geometry.

But to us humans it will seem to be like a triangle with the middle parts of the lines bent in toward the center. Thus the angles at the tips will be less than what they are for usual Euclidean triangles. So, if you add all three, the sum will be less than 180 degrees, the sum for a Euclidean triangle.

If you have trouble visualizing paths like this on a saddle, try doing it on a sphere (like the surface of the earth). This will be spherical geometry, which has the opposite property: in spherical geometry, the sum of the angles of a triangle will be more than 180 degrees.

To see why, imagine the following route: start at the north pole, and go south along the prime meridian through Greenwich down to the equator. Now take a 90 degree turn and go $1 / 4$ of the way around the earth, about to the Galapagos islands off the coast of Ecuador. Turn 90 degrees and go due north to the north pole. You will arrive there and your arrival and departure paths from the pole will make a 90 degree angle. Thus you've traced out a "triangle" of three shortest paths, and each angle of that triangle is 90 degrees, for a total of 270: much more than 180 degrees. By choosing triangles carefully on the surface of the earth, you can make them have a sum of angles of any value up to 540 degrees.

## TRADURRE IN INGLESE LE FRASI SEGUENTI:

1. Cosa è un triangolo?
2. Quanto vale la somma degli angoli di un triangolo?
3. Su una superficie iperbolica la somma degli angoli di un triangolo è meno di 180 gradi.
4. Due rette sono parallele se non hanno punti in comune.
5. Su una superficie sferica tutte le rette si intersecano e quindi non ci sono rette parallele!
