INdAM meeting

Homological and Computational Methods in Commutative Algebra

A conference dedicated to Winfried Bruns on the occasion of his 70th birthday

Cortona, May 30 - June 3, 2016

Monday	May 30
09:00 - 09:45	**Registration**
Room:	"Aula Papacello" (blackboards)
09:45 - 10:00	**Opening of the Conference**
10:00 - 10:45	David Eisenbud, *Layered Resolutions of Cohen-Macaulay modules*
11:00 - 11.30	**Coffee Break**
11:30 - 12:15	Holger Brenner, *Symmetric differential signature*
12:30 - 13:30	**Lunch Break**
Room:	"Aula Papacello" (computer presentations)
15:00 - 15:45	Bernd Ulrich, *Rees algebras of grade three Gorenstein ideals*
16:00 - 16:45	Hidefumi Ohsugi, *Ehrhart series of fractional stable set polytopes*
17:00 - 17:30	**Coffee Break**
17:30 - 18:15	Frank-Olaf Schreyer, *Unirational moduli, Hurwitz spaces and random curves*
Tuesday	May 31
Room:	"Aula Papacello" (blackboards)
09:00 - 10:00	Anurag Singh, *Determinantal rings and arithmetic rank*
10:15 - 11:00	Hubert Flenner, *Cancellation of surfaces*
11:00 - 11.30	**Coffee Break**
11:30 - 12:15	Marc Chardin, *Residual intersections*
12:30 - 13:30	**Lunch Break**
Room:	"Aula Papacello" (computer presentations)
15:00 - 15:45	Alfio Ragusa, *Algebras with the Weak Lefschetz Property*
16:00 - 16:45	Matteo Varbaro, *F-thresholds of determinantal objects*
17:00 - 17:30	**Coffee Break**
17:30 - 18:15	Volkmar Welker, *TBA*

Wednesday	June 01
Room:	"Aula Papacello" (blackboards)
09:00 - 10:00	Srikanth Iyengar, *Bruns' work on homological aspects of commutative algebra*
10:15 - 11:00	Ragnar-Olaf Buchweitz, *Ulrich Modules over Hesse Cubics*
11:00 - 11.30	**Coffee Break**
11:30 - 12:15	Luchezar Avramov, *Betti tables over short Gorenstein rings*
12:30 - 13:30	**Lunch Break**
14:00 - 23:00	**Excursion and Social Dinner**

Thursday	June 02
Room:	"Aula Papacello" (computer presentations)
09:00 - 10:00	Michael Joswig, *Software, Lattice Points and Free Sums of Polytopes*
10:15 - 11:00	Melvin Hochster, *Lim Cohen-Macaulay sequences of modules and content of local cohomology*
11:00 - 11.30	**Coffee Break**
11:30 - 12:15	Ezra Miller, *Homological algebra for real multigraded modules*
12:30 - 13:30	**Lunch Break**
Room:	"Aula Papacello" (blackboards)
15:00 - 15:45	Craig Huneke, *Symbolic Topologies in Complete Local Domains*
16:00 - 16:45	Milena Hering, *The F-splitting ratio of a toric variety*
17:00 - 17:30	**Coffee Break**
17:30 - 18:15	Giulio Caviglia, *Decidability of Bounds for Projective Dimension and Castelnuovo-Mumford Regularity*

Friday	June 03
Room:	"Aula Papacello" (computer presentations)
09:00 - 10:00	Jürgen Herzog, *To Winfried's 70th birthday - his life and his work*
10:15 - 11:00	Ngô Việt Trung, *Castelnuovo-Mumford regularity and Ratliff-Rush closure*
11:00 - 11.30	**Coffee Break**
Room:	"Aula Papacello" (blackboards)
11:30 - 12:15	Hal Schenck, *The Weak Lefschetz property for quotients by Quadratic Monomials*
12:30 - 13:30	**Lunch Break**
Room:	"Aula Papacello" (blackboards)
15:00 - 15:45	Takayuki Hibi, *A Gröbner basis characterization for chordal comparability graphs*
16:00 - 16:45	Uwe Nagel, *Equivariant Hilbert Series*

Abstracts

Betti tables over short Gorenstein rings rings

Luchezar Avramov
University of Nebraska, USA

Abstract: Necessary and sufficient conditions will be presented for a matrix of natural numbers to be the Betti table of a graded module over a standard graded Gorenstein ring R with $R_3 = 0$. The talk is based on joint work with Courtney Gibbons and Roger Wiegand.

Symmetric differential signature

Holger Brenner
Universität Osnabrück, Germany

Abstract: We introduce the symmetric signature which is a possible characteristic zero replacement for the F-signature. We prove that for invariant rings of a small finite group G the differential symmetric signature is $1/|G|$. This is based on joint work with Alessio Caminata.

Ulrich Modules over Hesse Cubics

Ragnar-Olaf Buchweitz
University of Toronto, Canada

Abstract: Given a smooth cubic in Hesse form in the projective plane, we will (discuss how to) find all Ulrich modules over its homogeneous coordinate ring. This is joint work with Alexander (Sasha) Pavlov.

Decidability of Bounds for Projective Dimension and Castelnuovo-Mumford Regularity

Giulio Caviglia
Purdue University, USA

Abstract: A question of Stillman asks: Is there a bound for the projective dimension of every homogeneous ideal generated in a polynomial ring, which depends solely on the knowledge of the degrees of the generators, and not on the number of variables? In joint work with A. Banerjee, we show that for every fixed degree sequence, the correctness of a conjectural bound can be proved by an implementable algorithm.

Residual intersections

Marc Chardin
Université Pierre et Marie Curie, France

Abstract: Residual intersection is a generalization of liaison that was introduced by Artin and Nagata, and received important developments in works of Huneke and Ulrich, notably. We will present ongoing work with Naéliton and Tran showing the expected behavior for residual intersections of strongly Cohen-Macaulay ideals (and for part of the results, slightly more generally) without assuming any hypothesis on the local number of generators; such a type of condition was present in most results, before the work of Hassanzadeh that we build on.

Layered Resolutions of Cohen-Macaulay modules

David Eisenbud
MSRI and UC Bekeley, USA

Abstract: Ill describe the layered resolution" of a Cohen-Macaulay module M over a regular ring S, which depends on the choice of a maximal regular sequence f in the annihilator of M, and is minimal in case M is a high syzygy over $S/(f)$. The method also yields a resolution over $S/(f)$, which again is minimal in the case of a high syzygy. This is joint work with Irena Peeva.

Cancellation of surfaces

Hubert Flenner
Ruhr-Universität Bochum, Germany

Abstract: The famous Zariski Cancellation Problem asks as to when the existence of an isomorphism $X \times \mathbb{A}^1 \cong Y \times \mathbb{A}^1$ for (affine) algebraic varieties X and Y implies that $X \cong Y$. If Y is the affine plane then this holds by a result of Miyanishi-Sugie. Danielewski gave the first examples of surfaces for which we have non-cancellation. By a result of Bandman and Makar-Limanov, cancellation for surfaces always holds if the surfaces do not admit $\mathbb{A}^1$-fibrations over a curve. Given two $\mathbb{A}^1$-fibered surfaces over the same affine curve B with reduced fibers, we provide a criterion, expressed in terms of linear equivalence of certain divisors, as to when the corresponding cylinders are isomorphic over B. (Report on ongoing work with S. Kaliman and M. Zaidenberg).

The F-splitting ratio of a toric variety

Milena Hering
Edinburgh University, Scotland

Abstract: The Frobenius morphism is a useful tool in the study of commutative rings and algebraic varieties. One of its uses is to give a measurement of how bad the singularities of a ring are. This measurement is called the the F-splitting ratio, which agrees with the F-signature for normal rings. I will give give an introduction to these notions and present the computation of the F-splitting ratio of a seminormal toric ring. This is joint work with Kevin Tucker.

A Gröbner basis characterization for chordal comparability graphs

Takayuki Hibi
Osaka University, Japan

Abstract: My talk is based on `arXiv:1602.00285` which is a joint work with Hidefumi Ohsugi. A certain toric ideal arising from a set of multichains of a finite partially ordered set (poset) is studied. It is shown that the comparability graph of a finite poset is chordal if and only if the toric ideal possesses a quadratic Gröbner basis. A strong perfect elimination ordering of a strongly chordal graph plays an important role.

Lim Cohen-Macaulay sequences of modules and content of local cohomology

Melvin Hochster
University of Michigan, USA

Abstract: The work described is joint with Bhargav Bhatt and Linquan Ma.

Let R be a complete local domain. We define a countably infinite sequence of modules M_n to be lim Cohen-Macaulay if for some (equivalently, every) system of parameters x, $h_i(x, M_n)/h_0(x, M_n)$ approaches 0 as n approaches infinity for all $i > 0$, where $h_i(x, M_n)$ denotes the length of the i-th Koszul homology module of M. We show that such sequences exist in characteristic $p > 0$. Whether they exist in mixed characteristic is an open question. The existence of such sequences in mixed characteristic implies the positivity of Serre intersection multiplicities in mixed characteristic, and also yields a closure operation that generalizes tight closure and is sufficient to prove the existence of big Cohen-Macaulay modules. We also show that a weaker notion of lim Cohen-Macaulay sequence implies the direct summand conjecture, using ideas related to theory of content of local cohomology. Linquan Ma has used these ideas to settle Lech's conjecture in dimension 3 in equal characteristic.

Symbolic Topologies in Complete Local Domains

Craig Huneke
University of Virginia, USA

Abstract: This talk will focus on the following conjecture: if R is a complete local domain then there is a constant C such that for every n and for every prime P, the (Cn)-th symbolic power of P is contained in the the nth power of P. Recent advances by Daniel Katz and the speaker will be discussed as well as some history behind the problem.

Bruns' work on homological aspects of commutative algebra

Srikanth Iyengar
University of Utah, USA

Abstract: The purpose of this talk is to give an overview of some of Winfried Bruns' many contributions to commutative algebra that have a 'homological' flavour.

Software, Lattice Points and Free Sums of Polytopes

Michael Joswig
Technischen Universität Berlin, Germany

Abstract: The free sum of two polytopes is a basic operation in polytope theory; it is dual to the product. Formulae for the Ehrhart series of free sums received considerable attention. The connection with triangulations of lattice polytopes will be discussed. Mathematical software systems (such as normaliz and polymake) will play a role, too.

Homological algebra for real multigraded modules

Ezra Miller
Duke University, USA

Abstract: An introduction to how computational topologists view multigraded commutative algebra, and how combinatorial commutative algebra responds by providing data structures for real multiparameter persistence modules. Homological constructions and results include encodings by finite posets, presentations by generators and cogenerators, and syzygy theorems. Many of these are new even for finitely generated $\mathbb{Z}^n$-graded modules over polynomial rings.

Equivariant Hilbert Series

Uwe Nagel
University of Kentucky, USA

Abstract: Ideals in polynomial rings in countably many variables that are invariant under a suitable action of a symmetric group or the monoid of strictly increasing functions arise in various contexts. We study such an ideal using an ascending chain of invariant ideals. We establish that the associated equivariant Hilbert series is a rational function in two variables. This is used to prove that the Krull dimensions and multiplicities of ideals in such an invariant filtration grow eventually linearly and exponentially, respectively. Furthermore, we determine the terms that dominate this growth. This may also be viewed as a method for assigning new asymptotic invariants to a homogenous ideal in a noetherian polynomial ring. This is based on joint work with Tim Römer.

Ehrhart series of fractional stable set polytopes

Hidefumi Ohsugi
Kwansei Gakuin University, Japan

Abstract: The fractional stable set polytope FRAC(G) of a simple graph G with d vertices is a rational polytope that is the set of nonnegative vectors $(x_1, \ldots, x_d)$ satisfying $x_i + x_j \leq 1$ for every edge $\{i, j\}$ of G. The fractional stable set polytope FRAC(G) is a lattice polytope if and only if G is bipartite. In this talk, we show that (i) The δ-vector of a lattice polytope 2FRAC(G) is alternatingly increasing; (ii) The Ehrhart ring of FRAC(G) is Gorenstein; (iii) The coefficients of the numerator of the Ehrhart series of FRAC(G) are symmetric, unimodal and computed by the δ-vector of 2FRAC(G). This is a joint work with Ginji Hamano and Takayuki Hibi.

Algebras with the Weak Lefschetz Property

Alfio Ragusa
Università di Catania, Italy

Abstract: The talk is a survey on some joint works, with Giuseppe Zappalà and Giuseppe Favacchio, on algebras with the Weak Lefschetz Property and on Weak Lefschetz sequences. From this investigation it rises up in a natural way the definition of Algebra with a Betti Weak Lefschetz Property and we study this new property producing some example of algebra with such a property.

The Weak Lefschetz property for quotients by Quadratic Monomials

Hal Schenck
University of Illinois, USA

Abstract: Michałek–Miró-Roig recently gave beautiful geometric characterization of quotients by ideals generated by quadratic or cubic monomials, such that the multiplication map by a general linear form fails to be injective in the first nontrivial degree. Their work was motivated by a conjecture of Ilardi connecting the failure to Laplace equations and classical results of Togliatti on osculating planes, and work of Mezzetti-Miró-Roig-Ottaviani. We investigate the Weak Lefschetz property for quotients by quadratic monomial ideals, explaining failure of the Weak Lefschetz Property for some cases not covered by previous work (joint with J. Migliore, U. Nagel)

Unirational moduli, Hurwitz spaces and random curves.

Frank-Olaf Schreyer
Universität des Saarlandes, Germany

Abstract: In the talk I will report on computer aided unirationality proofs using various models of curves. The goal is to point out the specific difficulty in the constructions for each of the different approaches.

Castelnuovo-Mumford regularity and Ratliff-Rush closure

Ngô Viêt Trung
Institute of Mathematics, IMH-VAST, Vietnam

Abstract: We establish strong relationships between the Castelnuovo-Mumford regularity and the Ratliff-Rush closure of ideals. Our results can be used to study the Castelnuovo-Mumford regularity of the Rees algebra and of the fiber ring. As an application, we settle a conjecture of Eisenbud and Ulrich for large classes of monomial ideals in two variables. This is a joint work with M.E. Rossi and D.T. Trung..

Rees algebras of grade three Gorenstein ideals

Bernd Ulrich
Purdue University, USA

Abstract: We study defining ideals of Rees algebras, mainly Rees algebras of grade three Gorenstein ideals. This is joint work with Andy Kustin and Claudia Polini.

F-thresholds of determinantal objects

Matteo Varbaro
Università di Genova, Italy

Abstract: Given a matrix X of indeterminates over a field of positive characteristic K, I will outline a general method to compute the F-thresholds of ideals arising by taking sums and products of determinant ideals in $K[X]$. One key point for the method is provided by a result of W. Bruns describing primary decompositions of integral closures of powers of determinant ideals.