$$a^{x+y} = a^x a^y \qquad .$$

$$a^{x-y} = \frac{a^x}{a^y} \qquad .$$

$$(a^x)^y = a^{xy} \qquad .$$

$$\log_a(xy) = \log_a(x) + \log_a(y) \qquad x > 0 \quad y > 0 \qquad .$$

$$\log_a(\frac{x}{y}) = \log_a(x) - \log_a(y) \qquad x > 0 \quad y > 0$$

$$\log_a(x^b) = b \log_a(x) \qquad x > 0 \quad y \in \mathbf{R} \qquad .$$

Proprierà dell'esponenziale e dei logaritmi.

$$\lim_{n\to\infty} x^n = \begin{cases} 0 & \text{se } |x| < 1 \\ 1 & \text{se } x = 1 \\ +\infty & \text{se } x > 1 \\ \text{non esiste} & \text{se } x \leq -1 \end{cases}$$

Limite della successione $\{x^n\}$

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_o$$

$$Q(x) = b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_o$$

$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = \begin{cases} \frac{a_n}{b_m} \cdot (+\infty) & n > m \\ 0 & n < m \\ \frac{a_n}{b_n} & n = m \end{cases}$$

Limite a $+\infty$ di funzioni razionali

$\lim_{x\to+\infty} x^\alpha = +\infty \qquad \alpha > 0$	$\lim_{x\to+\infty} x^\alpha = 0 \qquad \alpha < 0$
$\lim_{x\to+\infty} a^x = +\infty$	$\lim_{x\to-\infty} x^a = 0 \qquad a > 1$
$\lim_{x\to+\infty} a^x = 0$	$\lim_{x\to-\infty} a^x = +\infty \qquad a < 1$
$\lim_{x\to+\infty} \log_a(x) = +\infty$	$\lim_{x\to0+} \log_a(x) = -\infty \qquad a > 1$
$\lim_{x\to+\infty} \log_a(x) = -\infty$	$\lim_{x\to0+} \log_a(x) = +\infty \qquad a < 1$

Tavola I Limiti di funzioni elementari

$\lim_{x\to0} \frac{\sin(x)}{x} = 1$	$\lim_{x\to0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$
$\lim_{x\to0} \left(1+\frac{a}{x}\right)^x = e^a \qquad a \in \mathbf{R}$	
$\lim_{x\to0} (1+x)^{\frac{1}{x}} = e \qquad a \in \mathbf{R}$	
$\lim_{x\to0} \frac{a^x-1}{x} = \log(\mathrm{a}) \qquad a > 0$	
$\lim_{x\to0} \frac{\log_a(1+x)}{x} = \log_a(\mathrm{e}) \qquad a > 0$	
$\lim_{x\to0} \frac{(1+x)^\alpha-1}{x} = \alpha \qquad \alpha \in \mathbf{R}$	

Tavola II Limiti notevoli.

$$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\text{arcsin(x)} = x + \frac{1}{2 \cdot 3} x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} x^5 \dots + \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{(2n+1)} + o(x^{2n+2})$$

$$\text{arctg(x)} = x - \frac{x^3}{3} + \frac{x^5}{5} \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)} + o(x^{2n+2})$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots + (-1)^{n-1} \frac{x^n}{n} + o(x^n)$$

$$\text{tg}(x) = x + \frac{x^3}{3} + \frac{2x^5}{5} + \frac{17x^7}{315} + o(x^8)$$

$$(1+x)^\alpha = 1 + \alpha x + \binom{\alpha}{2} x^2 \dots + \binom{\alpha}{n} x^n + o(x^n)$$

$$\frac{1}{1-x} = 1 + x + x^2 \dots + x^n + o(x^n)$$

$$\frac{1}{1+x} = 1 - x + x^2 \dots + (-1)^n x^n + o(x^n).$$

$$\sqrt{1-x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o(x^3)$$

Tavola III Formule di Mac Laurin con resto di Peano di funzioni elementari.

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-k+1)}{k!}$$

$$k \in \mathbf{N}, \alpha \in \mathbf{R}.$$

$$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!}\ e^c$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + (-1)^{n+1} \frac{x^{2n+2}}{(2n+2)!}\ \sin(c)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots + (-1)^n \frac{x^{2n}}{(2n)!} + (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)!}\ \sin(c)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots + (-1)^{n-1} \frac{x^n}{n} + (-1)^n \frac{x^{n+1}}{n+1}\ \frac{1}{(1+c)^{n+1}}$$

$$(1+x)^\alpha = 1 + \alpha x + \binom{\alpha}{2} x^2 \dots + \binom{\alpha}{n} x^n + \binom{\alpha}{n+1} (1+c)^{\alpha-n-1} x^{n+1}$$

Tavola IV Formule di Mac Laurin con resto di Lagrange di funzioni elementari