Detection and Description of Scale Invariant Interest Points with Shearlets

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***Abstract*—Shearlets are a relatively new directional multi-scale framework for signal analysis, which have been shown effective to enhance signal discontinuities such as edges and corners at multiple scales. In this paper we address the problem of detecting and describing blob-like features in the shearlets framework. We derive a measure which is very effective for blob detection and closely related to the Laplacian of Gaussian. We demonstrate the measure satisfies the perfect scale invariance property in the continuous case. In the discrete setting, we derive algorithms for blob detection and feature point description. Finally, we report an experimental evidence that our method is very suitable to deal with compressed and noisy images, thanks to the sparsity property of shearlets.**

I. INTRODUCTION

Feature detection consists in the extraction of perceptually interesting low-level features (edges, ridges, corners or blobs) over an image, in preparation of higher level processing tasks. In the last decade scale-space theory has provided an effective framework for detecting features at multiple scales and for devising scale invariant image descriptors. Blob-like features are local key points where the image signal is approximately uniform which have been employed primarily in image matching and image recognition. In early works they have been enhanced through the Laplacian of the Gaussian (LoG) operator [1]. Later, difference of Gaussians (DoG) has been introduced as an efficient approximation of the Laplacian [2], while the Hessian determinant [1] was suggested as an alternative operator with a higher sensitivity and better invariance properties. Computationally efficient variants have also been devised [3]. Since feature detection often precedes feature matching, local features need to be associated with an appropriate descriptor. For a reliable feature matching, it is important to identify a descriptor able to deal with geometric transformations, illumination changes, and the presence of noise. Therefore over the years there has been a lot of work in devising feature descriptors able to address different types of variations [2]–[5].

Unsurprisingly, image feature detection at multiple scales has also been addressed in the context of wavelet theory [6]–[8]. This framework allows for a natural derivation of the feature scale [6], [8] and for the design of perfect scale-invariant measurements [9]. Also it provides an optimally sparse representation, very effective in the presence of noise. In the special case the mother wavelet is the derivative of the Gaussian, the wavelet transform is equivalent to the scale-space representation [6].

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In recent years there have emerged a large class of representations with a further sensitivity to directional information than classical wavelets. Here it is worth mentioning directional wavelets [10], contourlets [11], curvelets [12], and shearlets [13].

In this paper we focus on shearlets representation and we show how the use of shearlet coefficients may enhance blob structures in an image. Indeed, shearlets enjoy different interesting properties which are meaningful to feature detection and description:

1) For shearlets two consecutive scales are related by an anisotropic dilation with ratio $1/\sqrt{2}$ that provides an optimal sparse representation up to a log factor for natural images [14].

2) The shearlet coefficients directly encode directional information, unlike scale-space representations and traditional wavelets where one could derive directional information only as a post-processing step.

3) Shearlets provide an optimal sparse representation for two-dimensional signals having singularity along curves [14]. On the contrary, the coefficients of the noise are uniformly distributed over all the components. Hence, an accurate processing of the shearlet coefficients ensures both a sparse representation stable under compression and an effective denoising without adding artifacts [15].

4) In contrast to the scale-space approaches, with shearlets we have a large choice of admissible templates allowing to tune the shearlet transform to specific applications, e.g, the Gaussian derivative to locate edges or corners as in [16], [17], or the Mexican hat to analyze blob structures or ridge points.

5) Shearlets also appear to have a potential in providing meaningful descriptions [18], [19], although this capability has not been largely explored so far.

In this paper we leverage the sparsity and directional sensitivity of shearlets to design a robust algorithm for detecting and describing blob-like features in images. The use of a common underlying theory allows us to develop a detection-description pipeline which requires one main computation step only, i.e. the shearlet transform.

In order to develop a shearlet-based blob detector, we first provide an analysis of the perfect scale invariance properties of shearlets in the continuous case, similar to the study carried out by Lindberg for the scale-space [20]. Then, we derive a discretized formulation of the problem, obtaining a discrete measure which will be the main building block of the blob detector.

We present an experiment on a set of images, where we underline the appropriateness of the method to address image matching at different compression and noise levels. In this specific aspect resides one of the main contribution of our work from the application standpoint: the sparsity properties of the shearlet transform are very appropriate to deal with noise and compression artifacts.

The paper is organized as follows. In Section II we review the shearlet transform. Section III provide the theoretical justifications of scale invariance for feature detection by shearlets. In Section IV we propose the shearlet-based blob detection algorithm, while the descriptor is introduced in Section V. Section VI evaluates the proposed methods for image matching at different compression and noise levels. Section VII is left to a final discussion.

II. THE SHEARLET TRANSFORM

A shearlet is generated by the dilation, shearing and translation of a function $\psi \in L^2(\mathbb{R}^2)$, called the *mother shearlet*, in the following way

$$\psi_{a,s,t}(x) = a^{-3/4}\psi\left(\begin{pmatrix} \frac{1}{a} & -\frac{s}{a} \\ 0 & \frac{1}{\sqrt{a}} \end{pmatrix}(x-t)\right) \tag{1}$$

where $t \in \mathbb{R}^2, a \in \mathbb{R}^+$ and $s \in \mathbb{R}$ are the translation, dilation and shearing parameters respectively. The anisotropic dilation a controls the scale of the shearlets, by applying a different dilation factor along the two axes. The shearing parameter s determines the orientation of the shearlets. The normalization factor $a^{-3/4}$ ensures that $\|\psi_{a,s,t}\| = \|\psi\|$, where $\|\psi\|$ is the norm in $L^2(\mathbb{R}^2)$. The *shearlet transform* $\mathcal{SH}(f)$ of a signal $f \in L^2(\mathbb{R}^2)$ is defined by

$$\mathcal{SH}(f)(a,s,t) = \langle f, \psi_{a,s,t}\rangle \tag{2}$$

where $\langle f, \psi_{a,s,t}\rangle$ is the scalar product in $L^2(\mathbb{R}^2)$.

In the classical setting the *mother shearlet* ψ is assumed to factorize in the Fourier domain as $\hat{\psi}(\omega_1,\omega_2) = \hat{\psi}_1(\omega_1)\hat{\psi}_2(\frac{\omega_2}{\omega_1})$ where $\hat{\psi}$ is the Fourier transform of ψ, ψ_1 is a one dimensional wavelet and $\hat{\psi}_2$ is any non-zero square-integrable function. Different approaches are proposed in [21], [22]. However, for sake of simplicity, in this work we consider only classical shearlets.

A. Digital Shearlets

Digital shearlets are defined by sampling continuous shearlet systems on a discrete subset of the space of parameters $\mathbb{R}_+ \times \mathbb{R}^3$ and by sampling the signal on a grid.

In the literature there are many different discretization schemes, see [22], [23]. In this work we adopt the Fast Finite Shearlet Transform (FFST) [24] which performs the entire shearlet construction in the Fourier domain. It is possible to choose as ψ_1 wavelets whose analytic form is given in the Fourier domain, whereas in [16] and [22] wavelets are associated with a multiresolution analysis.

In this scheme, the signal is discretized on a square grid of size N, which is independent on the dilation and shearing parameter, whereas the scaling, shear and translation parameters are discretized as

$$a_j = 2^{-j}, \quad j = 0,\dots,j_0-1,$$
$$s_{j,k} = k2^{-j/2}, \quad -\lfloor 2^{j/2}\rfloor \le k \le \lfloor 2^{j/2}\rfloor,$$
$$t_m = \left(\frac{m_1}{N}, \frac{m_2}{N}\right), \quad m \in \mathcal{I}$$

where j_0 is the number of considered scales and $\mathcal{I} = \{(m_1,m_2) : m_1,m_2 = 0,\dots,N-1\}$. With respect to the original implementation we use a dyadic scale 2^{-j} instead of 4^{-j} to reduce the difference among two consecutive scales, which is consistent with the discretization lattice in [22].

We conclude by observing that the computational complexity of the FFST is approximately $\mathcal{O}(N^2 \log N)$. A more detailed analysis can be found in [17].

III. SCALE SELECTION WITH SHEARLETS

According to Lindeberg [25], the formal definition of *scale selection* refers to the estimation of characteristic scales in image data and the automatic selection of locally appropriate scales in a scale-space representation. A particularly useful methodology for computing estimates of characteristic scales is by detecting local extrema over scales of differential expressions in terms of γ-normalized derivatives [1].

In this section, we show how shearlet coefficients can also detect the correct scale while providing directional information. In the second part of the section we discuss how we can obtain a measure of scale invariance in the discrete setting.

A. Scale Invariance in the Continuous Setting

Since the dilation defining the shearlets is not isotropic, we can not expect that the shearlet transform itself is invariant under (isotropic) scale changes. However, we will show how a related quantity has the perfect scale invariance property, as demonstrated by the following result, whose proof can be found in the appendix of [26].

Theorem 1. *The cumulative shearlet transform*

$$B[f](a,z) = a^{-5/4}\int_{\mathbb{R}} \mathcal{SH}(f)(a,s,az)ds, \tag{3}$$

with $a \in \mathbb{R}_+$ and $z \in \mathbb{R}^2$, is scale invariant, i.e. *for all $f \in L^2(\mathbb{R}^2)$*

$$B[f_\alpha](a,z) = B[f](\alpha^{-1}a,z) \tag{4}$$

where $f_\alpha(x) = f(x/\alpha)$ with $\alpha \in \mathbb{R}^+$.

As in the scale-space theory, for the $2D$ sinusoidal signals

$$f(x_1,x_2) = \cos(\alpha x_1)\cos(\beta x_2), \tag{5}$$

the cumulative shearlet transform can be explicitly computed and it is equal to

$$B[f](a,z) = \overline{\psi_2(0)\hat{\psi}_1(\frac{a\alpha}{2\pi})}f(az), \tag{6}$$

provided that ψ_1 is even, so that the maximum of the modulus over z is

$$B_{\max}[f](a) = |\psi_2(0)|\,|\hat{\psi}_1(\frac{a\alpha}{2\pi})|. \tag{7}$$

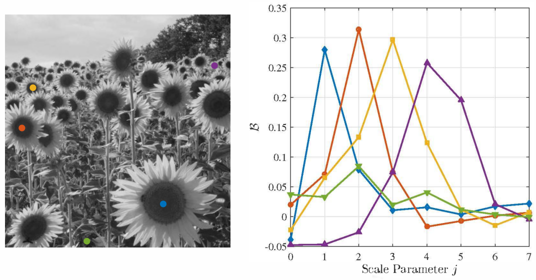


Fig. 1. $\mathcal{B}$ behavior across scales for different points, coherently color-coded.

By choosing $\hat{\psi}_1$ as the $1D$-*Mexican hat* wavelet

$$\hat{\psi}_1(\omega) = \omega^2 e^{-2\pi^2\omega^2}, \tag{8}$$

we can rewrite Eq. (7) as

$$B_{\max}[f](a) = \frac{|\psi_2(0)|}{4\pi^2}(a\alpha)^2 e^{-\frac{(a\alpha)^2}{2}}, \tag{9}$$

which shares the same behavior of the maximum of the LoG. Moreover, if we consider the shearlet coefficient and that both ψ_1 and ψ_2 are even, a computation as above shows that

$$\max_{t\in\mathbb{R}^2} |\mathcal{SH}(f)(a,s,t)| = a^{3/4}|\hat{\psi}_1(\frac{a\alpha}{2\pi})||\hat{\psi}_2(\frac{s+\beta\alpha^{-1}}{\sqrt{a}})|.$$

Since $\hat{\psi}_2$ is a bump function, fixed the scale a, the shearlet coefficients have a maximum around an interval centered at $s = -\beta/\alpha$. If $\hat{\psi}_2$ is a Gaussian bump and $\hat{\psi}_1$ is as in (8)

$$a^{-3/4} \max_{t\in\mathbb{R}^2} |\mathcal{SH}(f)(a,s,t)| = C(a\alpha)^2 e^{-\frac{(a\alpha)^2}{2}} e^{-\frac{(s+\beta\alpha^{-1})^2}{2a}}.$$

As we can notice, (9) is capable to produce perfect scale invariance for any combinations of the frequencies parameters α and β in the sinusoidal function f.

B. Scale Invariance in the Discrete Setting

In the previous section, we defined a scale invariant shearlet transform in the continuous setting. Now, let us formally define the discrete counterpart of Eq. (3), which we call the $\mathcal{B}$ measure.

Definition 1. *The $\mathcal{B}$ measure is the scale-normalized sum of the discrete shearlet transform coefficients across the shearing parameter.*

$$\mathcal{B}(m,j) = \frac{2^{\frac{5j}{4}}}{C_j}\sum_k \mathcal{SH}(\mathcal{I})(j,k,m), \tag{10}$$

where j,k,m are the discretized scaling, shearing and translation parameters.

The normalization factor C_j takes into account that for each scale j there is a different number of orientations.

We stress that the choice of ψ_1 and ψ_2 influences the type of local features that are enhanced by the shearlet transform. Thus, in order to detect blob features, as suggested by Eq. (9) we selected ψ_1 as the Mexican hat wavelet and ψ_2 as a smooth function with compact support whose analytic form is given in [24].

Fig. 1 illustrates the behavior of the $\mathcal{B}$ measure across scales for different key points of a real image. We consider in particular five locations corresponding to blob structures of different size and one texturized region. It is easy to observe that although there is no perfect scale invariance, the peaks are clearly visible and their position reflect the different spatial extents of the corresponding image structures.

IV. Blob Detection with Shearlets

In this section we deal with the problem of automatically detecting blobs and describe our *Shearlet Blob Detector (SBD)* algorithm. Similarly to the method proposed by Lowe to extract DoG features [2], our approach consists of different steps of measures, computation and refinement.

1) Accurate feature point localization: A location m at a certain scale j is recognized as a candidate keypoint if the function $\mathcal{B}(m,j)$, computed over a spatial $3\times3\times3$ (2D space $\times$ scales) neighborhood centered on m assumes a local extremum (maximum or minimum) in it and its value is above a threshold.

$$(\bar{m},\bar{j}) = \arg\max_{m,j}\min \text{ local } \mathcal{B}(m,j). \tag{11}$$

Then, the local extrema of the $\mathcal{B}$ function are interpolated in space and scale with the Brown and Lowe method [27] to reduce the effect of considering a limited number of scales.

2) Edge responses elimination: The function $\mathcal{B}$ has strong responses along edges, especially at fine scales. Therefore, in order to increase stability of the detected points, we need to eliminate the feature points that have high edge responses. That is, those detected points with high values of

$$\frac{1}{4\lfloor 2^{j/2}\rfloor}\sum_k \left(\mathcal{SH}(\mathcal{I})(j,k,m) - \mathcal{SH}(\mathcal{I})(j,k_{\max},m)\right)^2,$$

where $4\lfloor 2^{j/2}\rfloor$ is the total number of shearings for scale j, and $k_{\max}$ is the shearing with largest shearlet response,

$$k_{\max} = \arg\max_k |\mathcal{SH}(\mathcal{I})(j,k,m)|. \tag{12}$$

3) Accurate orientation assignment: In this step an orientation is assigned to each feature point. This is an important step in view of the computation of rotation invariant local feature descriptors. By means of the shearlet transform, the predominant orientation at a point m and scale j is easily obtained by finding the index $k_{\max}$ given by Eq. (12). However, the orientation estimation at coarse scales may have low accuracy since for small j a few shearings are employed. The effects can be attenuated by finding the extremum of a parabola fitted to $[k_{\max}, \mathcal{SH}(\mathcal{I})(j,k_{\max},m)]$ and its respective neighboring shears.

V. Feature Description with Shearlets

In this section we propose a local feature descriptor based on the shearlet transform, the Shearlet Local Description algorithm (SLD). The idea behind is to encode the shearlet coefficients computed from the SBD and thus complete the full detection-description pipeline with a single main computation, the shearlet transform in this case.

Given a feature point, our descriptor encodes the shearlet coefficients information from a square region centered on the feature point position m, scaled with respect its estimated scale j and rotated according to its predominant orientation θ.

1) Spatial sampling: For each detected feature point, we sample a regular grid of 24 points per side around m with a sampling step of $p = 2^{j\bullet - j}$, covering a length of $24p$. The grid, is then divided in 16 overlapped subregions of size $9p \times 9p$, hence using 81 shearlet coefficients. Notice that the overlap allows us to cope with small spatial feature point shifts.

2) Region rotation: In order to gain rotation invariance, the grid points are rotated according to the feature point main orientation θ. To maintain the rotation invariance, the shearing parameter k also has to be aligned according to θ.

3) Descriptor construction: The SLD descriptor collect statistics (mean and absolute mean) of the shearlets coefficients in each subregion. Moreover, the contributions of each subregion are weighted using a Gaussian and then concatenated to build the full descriptor of size $\mathbb{R}^{2\times c\times 16}$, where c is number of used shearings, usually $c = 4$. According to [3], the Gaussian weighting increase robustness towards geometric deformations and localization errors .

4) Descriptor normalization: Finally, in order to gain invariance to linear contrast changes, we normalized the descriptor to a unit vector, using the ℓ_2 normalization.

The computational cost of the proposed methods heavily depends on the computation of the shearlet transform. We are awere that the FFST implementation is not the most efficient. However, our choice is motivated by the possibility of changing modularly the mother wavelet ψ_1 to design different feature detection algorithms (edges, corners and now blobs). In future works we will consider more efficient alternatives like [13], [28], [29].

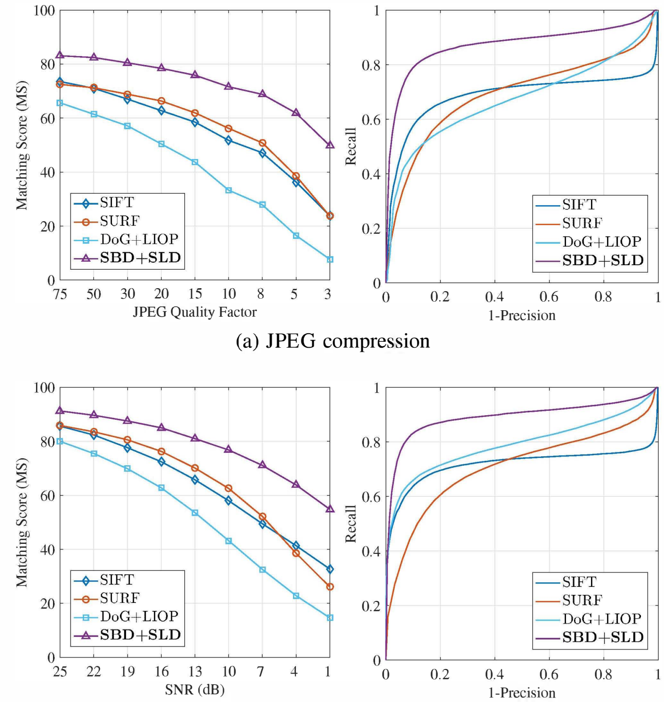
VI. Experimental Results

In this section we provide an experimental assessment of our full pipeline, detection plus description, to address image matching at different compression and noise levels.

The evaluation of our methods versus other state of the art detectors and descriptors following the *classical Mikolajczyk's protocol* [4] can be found in [26].

As anticipated in the introduction, a theoretical analysis proves that shearlet transform provides a sparse representation where the points of interest are associated with high coefficients, whereas the noise contribution is equally distributed. Hence there is a theoretical guarantee that descriptors based on shearlets thresholding are stable under compression and noise. This section is aimed to assess this behavior on real data. We discuss the use of our full pipeline (SBD+SLD) on a dataset of images, and consider the problem of matching images characterized by different levels of compression and noise. We evaluate our results on the *INRIA Copydays* dataset[1], which

[1]The datasets is available at https://lear.inrialpes.fr/~jegou/data.php



(a) JPEG compression

(b) Gaussian noise corruption

Fig. 2. Comparison of blob detectors with their respective descriptor on the *INRIA Copydays dataset*. Left: matching score against amount of compression (a) and noise corruption (b). Right: recall vs 1-precision curve between untransformed and 15 QF compressed (a) and 13 dB of SNR noise corrupted images.

contains 157 natural images that are progressively compressed, from 3 (very low quality) to 75 (typical web quality) quality factor (QF). For the evaluation in noisy environments, the images were also progressively corrupted with Gaussian noise.

We compare our method with the SIFT [2] and SURF [3] methods (DoG+SIFT and fastHessian+SURF, respectively), along with the DoG+LIOP [5] detector-descriptor combination. For the evaluation, we consider the *Matching Score* (MS) [30] which is the ratio between the number of correct matches and the number of detected features. Fig. 2 (left) shows the comparison, where the matching superiority of our approach can be appreciated in both JPEG compression (a) and noise corruption (b).

As a further evidence, we also provide the *recall* (number of correct matches / number of correspondences) vs *1-precision* (number of false matches / number of matches) curve (see Fig. 2, right) obtained when matching the (untransformed) images with the compressed instances (15 quality factor) in (a) and noisy instances (13 dB of Signal to Noise Ratio) in (b). Note that our approach consistently outperforms the competitors. For a visual impression of the overall performances on the entire dataset, average values, are reported in both figures.

The results we obtained are in good agreement with the theoretical intuition that shearlets are an appropriate choice in particular when dealing with noisy and compressed signals.

VII. Conclusions

In this paper we considered the shearlet representation as a multi-scale framework for the detection and the description of scale-invariant interest points. We first provided a comparative analysis of scale invariance in the shearlets domains. In the continuous case, we followed the reasoning proposed by Lindeberg [31] for scale-space. Then, we considered the discrete setting where we addressed the problem of detecting and describing blob-like features by means of the shearlet transform, exploiting it capability of embedding naturally both scale and orientation information. More specifically, we proposed a Shearlet Blob Detector (SBD) algorithm and a Shearlet Local Descriptor (SLD) algorithm, which we experimentally assessed for image matching considering a dataset of images affected by different degrees of noise or compression degradation. The results shows how our shearlet-based pipeline, which includes both detection and description, provided superior results to the SIFT and SURF methods.

In future works different shearlet transform alternatives will be worth investigating. In particular, compactly supported shearlets in the space domain [21] have been recently shown to have nice properties for edge detection [32] since they could allow us to capture effectively the spatial locality of image features. In addition, by using the 3D shearlet transform [33], we also have the interest of extending the proposed shearlet detectors to 3D signals and video image sequences (2D + time).

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