Metodi di regolarizzazione innovativi per dati telerilevati

Flavia Lenti, Claudio Estatico, Giuseppe Rodriguez, Ferdinando Nunziata, Maurizio Migliaccio

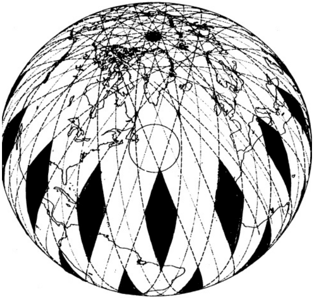
Dipartimento di Scienza e Alta Tecnologia,
Università degli Studi dell'Insubria

Outline

Outline

Earth Observation by microwave remote sensing

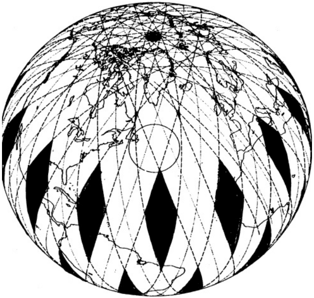
Microwave remote sensing from space-borne sensors plays a fundamental role in several remote sensing applications



- dense spatial coverage
- dense temporal coverage

Earth Observation by microwave remote sensing

Microwave remote sensing from space-borne sensors plays a fundamental role in several remote sensing applications



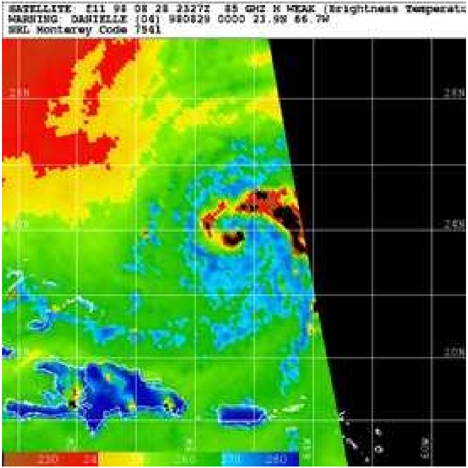
- dense spatial coverage
- dense temporal coverage

Radiometer and Scatterometer

Spatial Resolution: a limiting factor

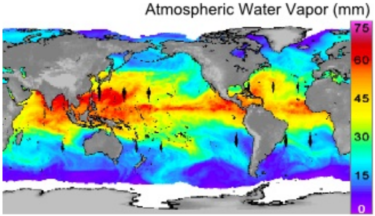
1. **Small but significant features are to be observed**

Thuderstorm



Spatial Resolution: a limiting factor

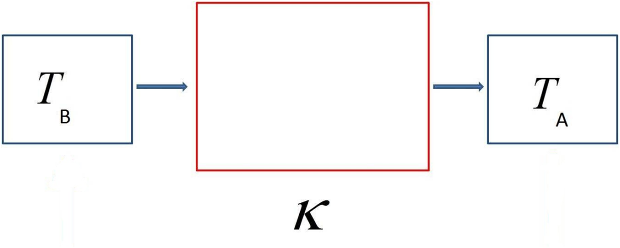
1. Small but significant features are to be observed
2. **Data fusion among various sensor channels**



Atmospheric water vapor maps generated by combining of 19V and 22V measurements

SSM/I channel's	3 dB Footprint	Spacing (km)
19.35V/H	69×43	25
22.235V	50×40	25
37.0V/H	37×28	25
85.5V/H	15×13	12.5

Measurements

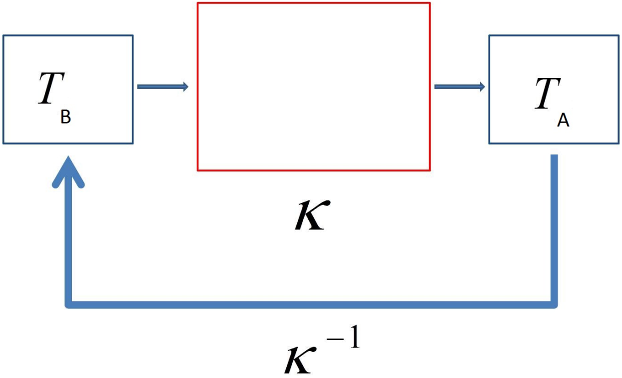


In formula:

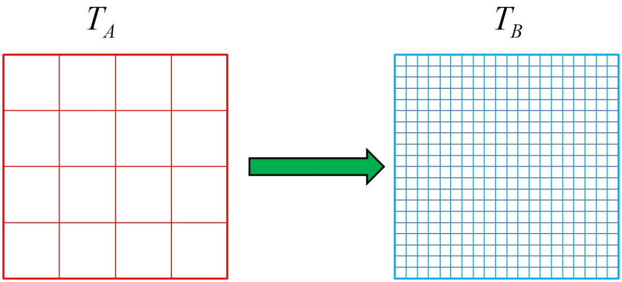
$$T_A(s,t) = \int \int_{\Omega} K(x,y,s,t) T_B(x,y) dxdy$$

Equation of Fredholm of the first kind

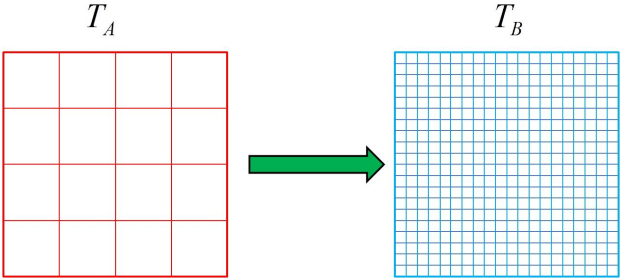
Inverse Problem

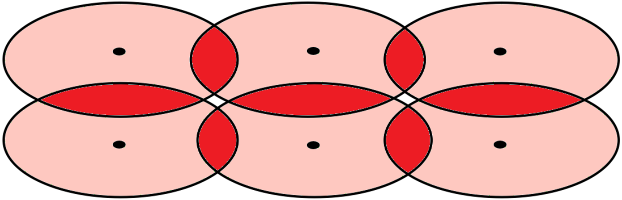


Resolution Enhancement



Resolution Enhancement





Outline

Discretization

The continuos model is mainly of theoretical interest since in pratice, data are collected at discrete values

- Data provided by the problem $\longrightarrow$ $B \in \mathbf{R}^{m_1 \times m_2}$ of observables data
- Discretization of the solution $\longrightarrow$ $X \in \mathbf{R}^{n_1 \times n_2}$ of unknown brightness field
- Discretization of the integral $\longrightarrow$ $L : \mathbf{R}^{m_1 \times m_2} \to \mathbf{R}^{n_1 \times n_2}$

$$\Downarrow$$

$$L(X) = B$$

Ill- conditioned problem

Approach to solution

$$L(X) = B$$

Regularization of the problem

- Vectorize X and B
  Cast the problem in standard matrix form
  Apply standard procedure

- Approach based on Fredholm integral equation with separable kernel.

Approach to solution

$$L(X) = B$$

Regularization of the problem

- Vectorize X and B
  Cast the problem in standard matrix form
  Apply standard procedure

- Approach based on Fredholm integral equation with separable kernel.

Approach to solution

L(X) = B

Regularization of the problem

- Vectorize X and B
  Cast the problem in standard matrix form
  Apply standard procedure

- Approach based on Fredholm integral equation with separable kernel.

$$K(x,y,s,t) = K_1(x,s)K_2(y,t)$$

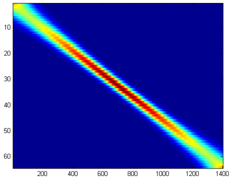
Discretization of Fredholm integral equation with separable kernel

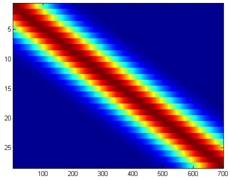
$$T_A(s,t) = \int_{\Omega_2} K_2(y,t) \int_{\Omega_1} K_1(x,s) T_B(x,y) dxdy$$

$$\Downarrow$$

$$A_1 X A_2^T = B$$

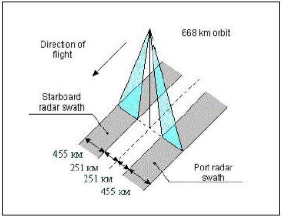
A_1 A_2





Matrix A_1

Linear Scansion

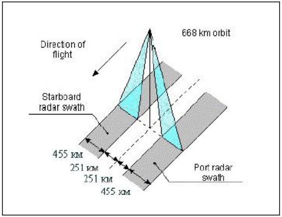


$$A_1(i,j) = Ce^{\left(-2\pi C_2\left(arctg\left(\frac{x_i}{r}\right) - arctg\left(\frac{s_j}{r}\right)\right)^2\right)}$$

with $x_i = \frac{1400}{n_1} * i,\ \ i = 1, ..., n_1$ and
$s_j = \frac{1400}{m_1} * j,\ j = 1, ..., m_1$

Matrix A_1

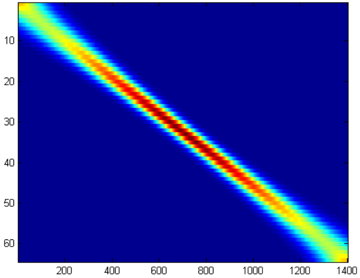
Linear Scansion



$$A_1(i,j) = Ce^{\left(-2\pi C_2 \left(arctg\left(\frac{x_i}{r}\right) - arctg\left(\frac{s_j}{r}\right)\right)^2\right)}$$

with $x_i = \frac{1400}{n_1} * i,\ i = 1, ..., n_1$ and $s_j = \frac{1400}{m_1} * j,\ j = 1, ..., m_1$

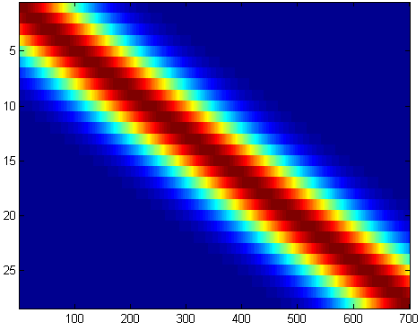
It is not a Toeplitz matrix but it is structured. By means of a coordinates trasformation, it can be transformed into a Toeplitz one.



Matrix A_2

$$A_2(i,j) = Ce^{-2\pi C_2 (y_i - t_j)^2}$$

with $y_i = \frac{700}{n_2} * i,\ i = 1, ..., n_2$ and $t_j = \frac{700}{m_2} * j,\ j = 1, ..., m_2$



It is a G-Toeplitz. It is:

$$A_2(i,j) = \mathbf{a}(\frac{n_2}{m_2} * i - j)$$

Reconstruction Algorithms

- Truncated Singular Value Decomposition
- Landweber Method in Hilbert Space
- Landweber Method in Banach Space

TSVD

$$A_1 X A_2^T = B$$

$$\Downarrow$$

$$A\mathbf{x} = \mathbf{b}$$

TSVD

$$A_1 X A_2^T = B$$

$$\Downarrow$$

$$A\mathbf{x} = \mathbf{b}$$

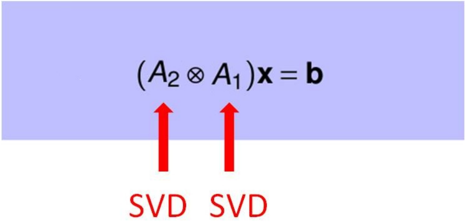
$$(A_2 \otimes A_1)\mathbf{x} = \mathbf{b}$$

TSVD

$$A_1 X A_2^T = B$$

$$\Downarrow$$

$$A\mathbf{x} = \mathbf{b}$$



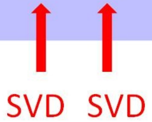
TSVD

$$A_1 X A_2^T = B$$

$$\Downarrow$$

$$A\mathbf{x} = \mathbf{b}$$

$$(A_2 \otimes A_1)\mathbf{x} = \mathbf{b}$$



TSVD solution

$$X_k = \sum_{j=1}^{k} v_{l(j),1} \frac{u_{l(j),1}^T B v_{h(j),2}}{\sigma_{l(j),1}\sigma_{h(j),2}} v_{h(j),2}^T$$

Landweber Method in Hilbert Space

Gradient Method

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \lambda \nabla \left(\frac{1}{2} \| A\mathbf{x}_k - \mathbf{b} \|_2^2\right) = \mathbf{x}_k - \lambda A^*(A\mathbf{x}_k - \mathbf{b})$$

Formally, A^* is the dual operator of A :

$$A : X \rightarrow Y, \quad A^* : Y^* \rightarrow X^*$$

where X^* and Y^* are the dual spaces of X and Y.

Landweber Method in Hilbert Space

Gradient Method

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \lambda\nabla(\frac{1}{2} \| A\mathbf{x}_k - \mathbf{b} \|_2^2) = \mathbf{x}_k - \lambda A^*(A\mathbf{x}_k - \mathbf{b})$$

Formally, A^* is the dual operator of A :

$$A : X \to Y, \quad A^* : Y^* \to X^*$$

where X^* and Y^* are the dual spaces of X and Y.

Riesz Theorem

If X is Hilbert space, X^* is isometrically isomorph to X

Landweber Method in Hilbert Space

Gradient Method

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \lambda \nabla (\frac{1}{2} \parallel A\mathbf{x}_k - \mathbf{b} \parallel_2^2) = \mathbf{x}_k - \lambda A^* (A\mathbf{x}_k - \mathbf{b})$$

Formally, A^* is the dual operator of A :

$$A : X \rightarrow Y, \quad A^* : Y^* \rightarrow X^*$$

where X^* and Y^* are the dual spaces of X and Y.

Riesz Theorem

If X is Hilbert space, X^* is isometrically isomorph to X

The Landweber method is well-defined in Hilbert Space

From Hilbert to Banach space

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \lambda A^*(A\mathbf{x}_k - \mathbf{b})$$

- $A\mathbf{x}_k - \mathbf{b} \in Y, A^* : Y^* \to X^* \Rightarrow A^*(A\mathbf{x}_k - b)$ is not correct
- $\mathbf{x}_k \in X, A^*\phi \in X^* \Rightarrow \mathbf{x}_k - A^*\phi$ is not correct

From Hilbert to Banach space

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \lambda A^*(A\mathbf{x}_k - \mathbf{b})$$

- $A\mathbf{x}_k - \mathbf{b} \in Y, A^* : Y^* \to X^* \Rightarrow A^*(A\mathbf{x}_k - b)$ is not correct
- $\mathbf{x}_k \in X, A^*\phi \in X^* \Rightarrow \mathbf{x}_k - A^*\phi$ is not correct

$$\mathbf{x}^*_{k+1} = \mathbf{x}^*_k - \lambda \Phi_A(\mathbf{x}_k, \mathbf{b})$$

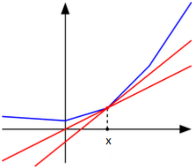
where $\Phi_A(\mathbf{x}_k, \mathbf{b})$ is the generalization of the gradient of the functional $\frac{1}{r} \| A\mathbf{x}_k - \mathbf{b} \|^r_Y$.

The key point: To generalize from Hilbert to Banach spaces we have to consider the so-called subdifferential

From Hilbert to Banach spaces (2)

Let f be a convex functional
$f : X \to \mathbf{R}$.
The subdifferential of f is the operator $\delta f : X \to 2^{X^*}$:

$$x^* \in \delta f(x) \leftrightarrow f(y) \le f(x) + x^*(y - x)$$



In Banach space l^p the subgradient of the functional $1/r \parallel A\mathbf{x}_k - \mathbf{b} \parallel_{l^p}^r$ is given by:

$$\delta(\frac{1}{r} \parallel A\mathbf{x}_k - \mathbf{b} \parallel_{l^p}^r) = \parallel A\mathbf{x}_k - \mathbf{b} \parallel_{l^p}^{r-p} A^*(|Ax_k - b|^{p-1} sgn(Ax_k - b)) =$$

$$= A^* J_r^{l^p}(A\mathbf{x}_k - \mathbf{b})$$

Landweber method in Banach space

$$Y = l^p \Rightarrow \mathbf{x}^*_{k+1} = \mathbf{x}^*_k - \lambda A^* J_r^{l^p}(A\mathbf{x}_k - \mathbf{b})$$

The duality map is a function which allows us to associate an element of a Banach Space X with an element of its dual X^* :

$$J_r^X : X \to 2^{X^*} \quad J_{r^*}^{X^*} : 2^{X^*} \to X$$

$$\text{with } r^* : \frac{1}{r} + \frac{1}{r^*} = 1.$$

$$\mathbf{x}_{k+1} = J_{r^*}^{X^*}(J_r^X(\mathbf{x}_k) - \lambda A^* J_r^Y(A\mathbf{x}_k - \mathbf{b}))$$

Landweber method in Banach space

$$Y = l^p \Rightarrow \mathbf{x}^*_{k+1} = \mathbf{x}^*_k - \lambda A^* J^{l^p}_r (A\mathbf{x}_k - \mathbf{b})$$

The duality map is a function which allows us to associate an element of a Banach Space X with an element of its dual X^* :

$$J^X_r : X \to 2^{X^*} \quad J^{X^*}_{r^*} : 2^{X^*} \to X$$

$$\text{with } r^* : \frac{1}{r} + \frac{1}{r^*} = 1.$$

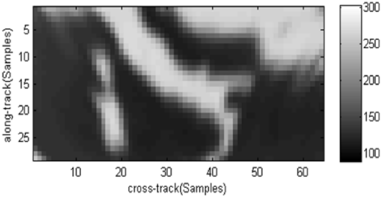
$$\mathbf{x}_{k+1} = J^{X^*}_{r^*}(J^X_r(\mathbf{x}_k) - \lambda A^* J^Y_r (A\mathbf{x}_k - \mathbf{b}))$$

$$A : l^p \to l^p \quad 1 \le p \le 2 \Rightarrow \mathbf{x}_{k+1} = J^{l^{p^*}}_{r^*} (J^{l^p}_r (\mathbf{x}_k) - \lambda A^* J^{l^p}_r (A\mathbf{x}_k - \mathbf{b}))$$

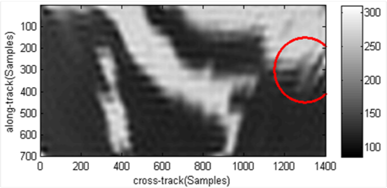
Outline

Real Radiometer Data

19 GHz V-polarized brightness temperature at non-enhanced resolution

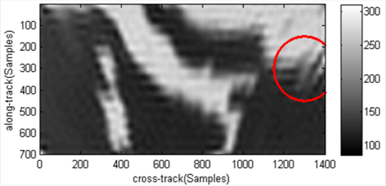


19 GHz V-polarized brightness temperature at enhanced resolution

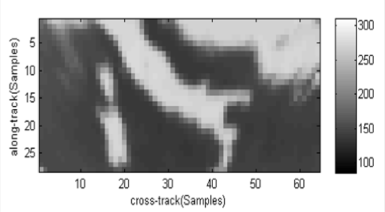


Real Radiometer Data

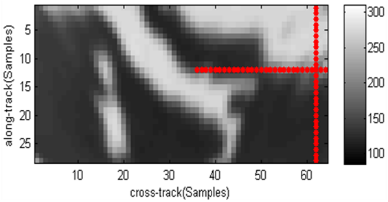
19 GHz V- polarized brightness temperature at enhanced resolution



37 GHz V-polarized brightness temperature

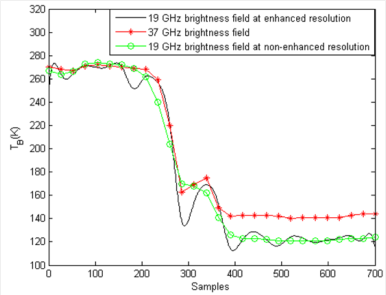


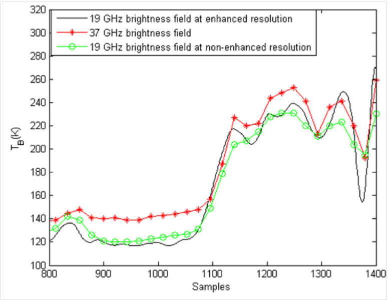
Real Radiometer Data



Along-track

Cross-track





Simulation Parameters for one-dimensional case

- Simulation of Reference Field/ Measurements
  SSM/I parameters:

  Area 1400 *km*
  Resolution 69 *km*
  Measurements 64

- Noise

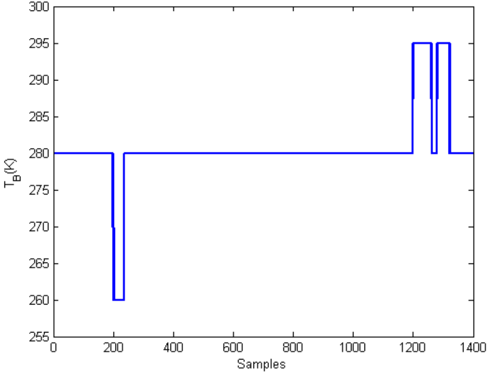
$$E = \mathcal{N}(0, 1.06)$$

- Antenna Gain

  Gaussian

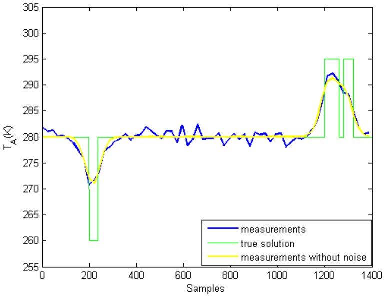
First Landweber experiment

The reference field includes three hot spots characterized by different brightness temperature and by different dimension. The simulated measurements are obtained by using Gaussian gain and a noise N (0,1.06).



First Landweber experiment

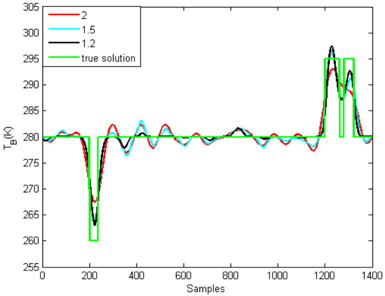
INPUT



Correlation coefficient

0.7626

RECONSTRUCTED FIELD

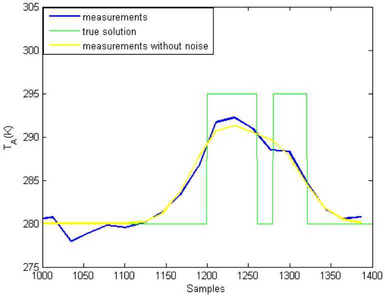


Correlation coefficient

p = 2	p = 1.5	p = 1.2
0.7745	0.8094	0.8455

First Landweber experiment

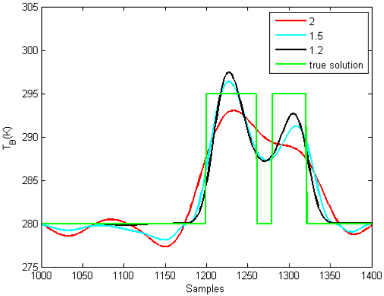
INPUT



Correlation coefficient

0.7626

RECONSTRUCTED FIELD

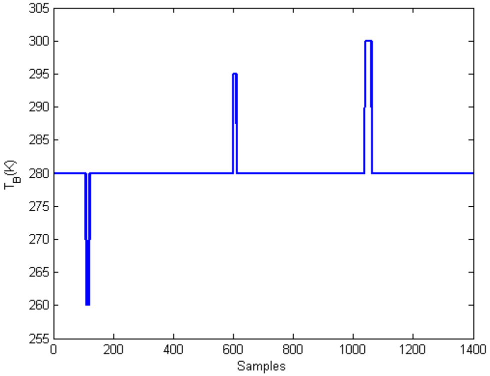


Correlation coefficient

p = 2	p = 1.5	p = 1.2
0.7745	0.8094	0.8455

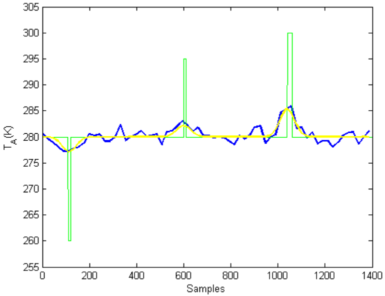
Second Landweber experiment

The reference field includes three hot spots characterized by very small dimension. The simulated measurements are obtained by using Gaussian gain and a noise N (0,1.06).



Second Landweber experiment

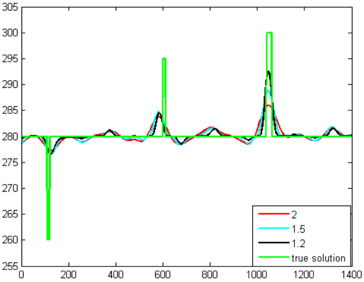
INPUT



Correlation coefficient

0.4108

RECONSTRUCTED FIELD

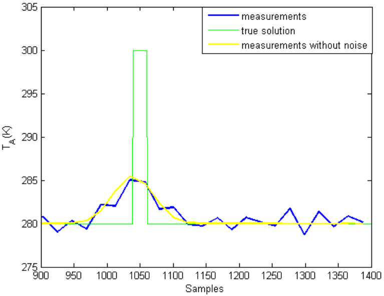


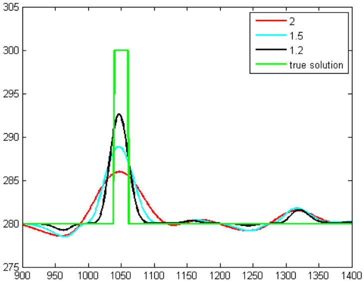
Correlation coefficient

p = 2	p = 1.5	p = 1.2
0.4584	0.5309	0.6186

Second Landweber experiment

INPUT	RECONSTRUCTED FIELD





Correlation coefficient

0.4108

Correlation coefficient

p = 2	p = 1.5	p = 1.2
0.4584	0.5309	0.6186

Thank you