Preface

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Directed Algebraic Topology is a recent field, deeply linked with Category Theory.

It studies 'directed spaces', where privileged directions are assigned in some way, so that paths and homotopies cannot generally be reversed. Ordered topological spaces are a basic structure of this kind; richer settings include locally ordered spaces and other topological structures mentioned in this volume, together with the classical combinatorial ones, i.e. simplicial and cubical sets, provided one does not ignore the choice of directions which they contain, as often done in algebraic topology.

Applications of this subject mostly deal with the analysis of concurrent processes and - recently - of rewriting systems; but its natural range should cover non reversible phenomena, in any domain. Links with noncommutative geometry have also been studied.

Directed spaces should be explored by means of algebraic tools which preserve, in some sense, the privileged directions of the original structures. For instance, the fundamental groups and the fundamental groupoid of a classical space should be replaced with the fundamental monoids and the fundamental category of a directed space; higher dimensional properties can be investigated with fundamental n-categories - possibly in a lax version.

Therefore, directed algebraic topology is linked with (higher) category theory in a clearer way than classical algebraic topology. And it can yield some geometric intuition to the study of weak or lax higher categories, which is also recent and complex.

This issue contains an introduction of mine to directed algebraic topology and its relationships with categories and higher categories. M. Raussen's paper investigates the complex notion of directed homotopy equivalence for directed spaces. E. Goubault and E. Haucourt deal with applications to concurrency, based on the fundamental category of an ordered space expressing the concurrent interactions. Y. Lafont's review of homological and homotopical tools in the theory of rewriting systems points to a new domain of application for directed algebraic topology. Finally, an article by E. Cheng is devoted to higher category theory, in connection with topological quantum field theories and cobordism.

Future developments will hopefully be able to further develop the interaction between directed algebraic topology and higher category theory, which was one of the main aims undertaken here.

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