Preface

This issue of "Applied Categorical Structures" is devoted to some aspects of Homotopy Theory, variously linked with the theory of categories. Rather than giving a summary of such aspects, which can be found in the Abstracts, we prefer to consider why these links are important and why Homotopy Theory and Category Theory are related by deep roots.

Concretely, it should be sufficient to mention the study of fundamental groupoids and their higher dimensional versions, which is forming a powerful link between these two subjects, as foreseen in Grothendieck's letter [Gr].

From a more formal point of view, higher dimensional homotopy theory and higher dimensional category theory seem even to move towards a sort of unification.

By a curious coincidence, this story began in the same year 1963, in both domains. Indeed, coherence problems - the core of both aspects - appeared in that year, independently, with Stasheff's works [St] on loop spaces and strongly homotopy associative differential algebras, on the one hand, and Mac Lane's paper [Ma] on coherence in monoidal categories, on the other hand. However, there was a striking difference in the two approaches, which is only now on the way of being solved: in the first case we have *one* operation with *unbounded* coherence conditions (the operation is associative up to a specified homotopy, which is coherent up to a specified 2-dimensional homotopy, and so on), while, in the second, monoidal categories have *two* operations with a sort of *truncated*, "sesquidimensional" coherence. After the general two-dimensional case, bicategories, developed by Benabou in 1967, higher dimensional category theory faced a long route, because of the difficulty of the coherence problems involved; only recently, various forms of weak n-category, for arbitrary n, and weak infinity category, have been proposed: a survey of ten such definitions can be found in Leinster [Le].

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