The facets of the cut polytope and the extreme rays of cone of concentration matrices of series-parallel graphs

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Joint work with Liam Solus and Caroline Uhler

Outline

1. Series-Parallel Graph
2. Three Convex Bodies
3. Facet-Ray Identification Property

Series-Parallel Graph

Definition

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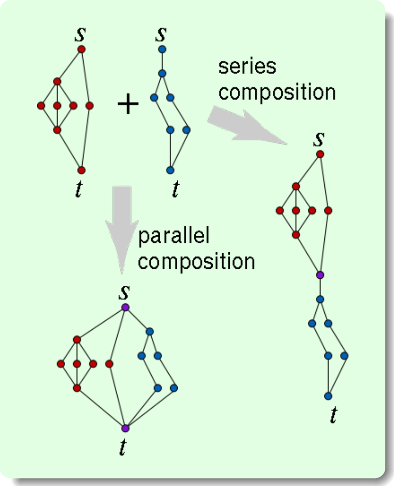

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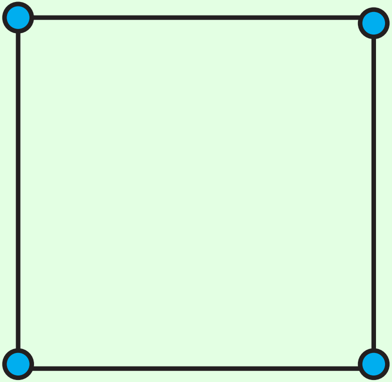

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Cut Polytopes

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A **cut** of the graph G is a bipartition of the vertices, (U, U^c), and its associated **cutset** is the collection of edges $\delta(U) \subset E$ with one endpoint in each block of the bipartition. To each cutset we assign a (± 1)-vector in $\mathbb{R}^E$ with a -1 in coordinate e if and only if $e \in \delta(U)$. The (± 1)-cut polytope of G is the convex hull in $\mathbb{R}^E$ of all such vectors.

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- Maximizing over the polytope $\text{cut}^{\pm 1}(G)$ is equivalent to solving the max-cut problem for G.
- The max-cut problem is known to be NP-hard.
- However, it is possible to optimize in polynomial time over a (often times non-polyhedral) positive semidefinite relaxation of $\text{cut}^{\pm 1}(G)$, known as an **elliptope**.

Cut Polytope for the 4-cycle: an example

$G := C_4$, identify $\mathbb{R}^{E(G)} \simeq \mathbb{R}^4$ by identifying edge $\{i, i+1\}$ with coordinate i for $i = 1, 2, 3, 4$. The cut polytope of G is the convex hull of $(-1, 1)$-vectors in $\mathbb{R}^4$ containing precisely an even number of -1's.

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Facets

$\text{cut}^{\pm 1}$ (G) is the 4-cube $[-1, 1]^4$ with truncations at the eight vertices containing an odd number of -1's with sixteen facets supported by the hyperplanes

$$\pm x_i = 1, \qquad \text{and} \qquad \langle v_T, x \rangle = 2,$$

where T is an odd cardinality subset of [4], and v_T is the corresponding vertex of $[-1, 1]^4$ with an odd number of -1's.

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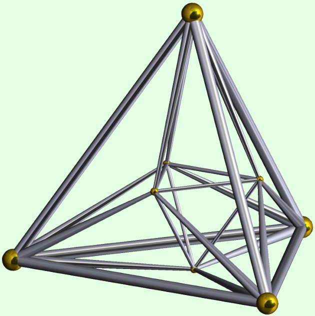

Schlegel diagram of the cut polytope for the 4-cycle.

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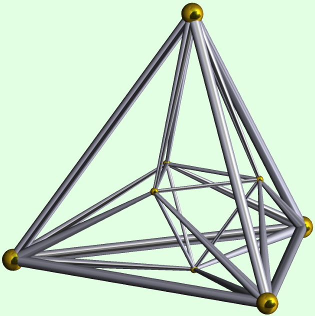

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Notes

It has 8 demicubes (tetrahedra) 8 tetrahedra as its facets.

Elliptopes

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Let $\mathbb{S}^p$ denote the real vector space of all real $p \times p$ symmetric matrices, and let $\mathbb{S}^p_{\succeq 0}$ denote the cone of all positive semidefinite matrices in $\mathbb{S}^p$. The ***p*-elliptope** is the collection of all $p \times p$ correlation matrices, i.e.

$$\mathcal{E}_p = \{X \in \mathbb{S}^p_{\succeq 0} | X_{ii} = 1 \text{ for all } i \in [p]\}.$$

The elliptope $\mathcal{E}_G$ is defined as the projection of $\mathcal{E}_p$ onto the edge set of G. That is,

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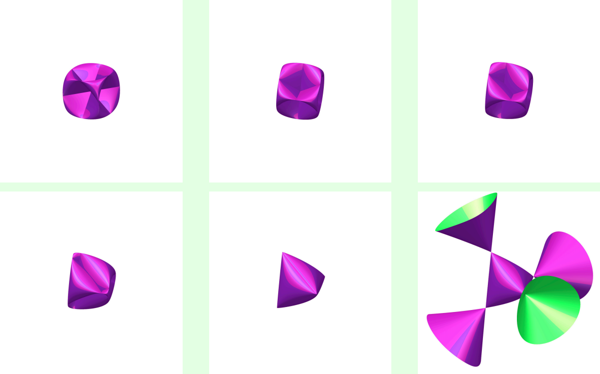
The elliptope $\mathcal{E}_G$ is a positive semidefinite relaxation of the cut polytope $\text{cut}^{\pm 1}$ (G), and thus maximizing over $\mathcal{E}_G$ can provide an approximate solution to the max-cut problem.

C_4-Elliptopes



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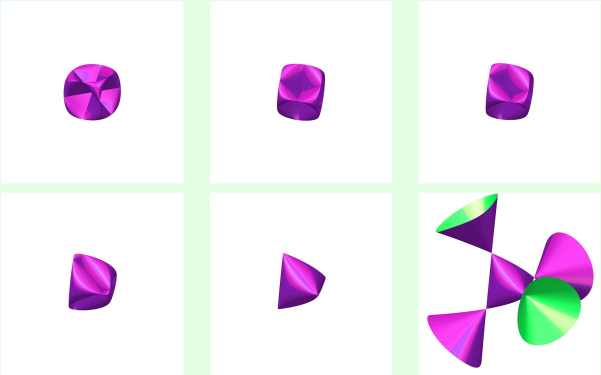

Level curves of the rank 2 locus of $\mathcal{E}_{C_4}$. The value of x_4 varies from 0 to 1 as we view the figures from left-to-right and top-to-bottom.

Cone of Concentration Matrices

Concentration Matrices

Consider the **Graphical Gaussian model** $N(\mu, \Sigma)$ where $\mu \in \mathbb{R}^p$ is the mean and $\Sigma \in \mathbb{R}^{p \times p}$ is the correlation matrix for the model. The concentration matrix of Σ is $K = \Sigma^{-1}$.

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A concentration matrix K is a $p \times p$ positive semidefinite matrices with zeros in all entries corresponding to nonedges of G.

Cone of Concentration Matrices

Let $\mathcal{K}_G$ is the set of all concentration matrices K corresponding to G. Then $\mathcal{K}_G$ is a convex cone in $\mathbb{S}^p$ called the **cone of concentration matrices**.

Applications

Goal

Want to show that the facets of $\text{cut}^{\pm 1}(G)$ identify extremal rays of $\mathcal{K}_G$ for any series parallel graph G.

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- The PD-completability problem would become easier for G with smaller sparsity order (i.e. where the max rank of an extremal ray is small).
- Our computations of the facets of $\text{cut}^{\pm 1}$ (G) for G series-parallel together with the proof of facet-ray identification tells us all these ranks are encoded nicely in the supporting hyperplanes of $\text{cut}^{\pm 1}$ (G).

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Theorem [Solus, Uhler, Y. 2015]

The dual body of the elliptope $\mathcal{E}_G$ is

$$\mathcal{E}_G^{\vee} = \{x \in \mathbb{R}^E \mid \exists\, X \in \mathcal{K}_G \text{ such that } X_E = x \text{ and } \operatorname{tr}(X) = 2\}.$$

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An immediate consequence of this theorem is that the extreme points in $\mathcal{E}_G^\vee$ are projections of extreme matrices in $\mathcal{K}_G$.

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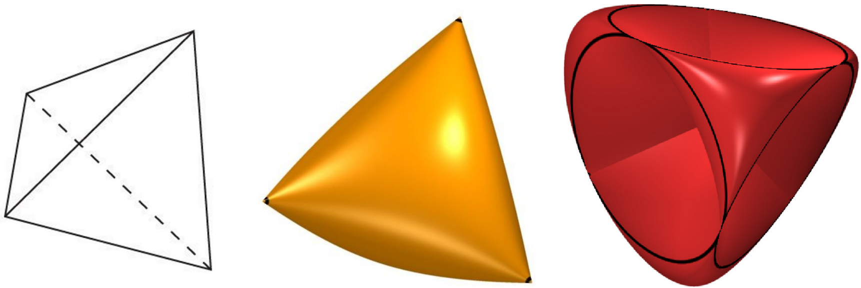
- Dually, the normal vectors to the facets of $\mathrm{cut}^{\pm 1}(G)$ are then extreme points of $\mathcal{E}_G^{\vee}$, and consequently projections of extreme matrices of $\mathcal{K}_G$.
- Thus, we can expect to find extremal matrices in $\mathcal{K}_G$ whose off-diagonal entries are given by the normal vectors to the facets of $\mathrm{cut}^{\pm 1}(G)$.

Example: 3-cycle



(a) $CUT^{\pm 1}(G)$ (b) $\mathcal{E}_G$ (c) $\mathcal{E}_G^V$

$$\mathrm{CUT}^{\pm 1}(G) = \mathrm{conv}((1,1,1),(-1,-1,1),(-1,1,-1),(1,-1,-1))$$

$$\mathcal{E}_G = \left\{ \begin{pmatrix} 1 & x_1 & x_3 \\ x_1 & 1 & x_2 \\ x_3 & x_2 & 1 \end{pmatrix} \succeq 0 \right\} \qquad \mathcal{E}_G^V = \left\{ \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} : \begin{pmatrix} a & y_1 & y_3 \\ y_1 & b & y_2 \\ y_3 & y_2 & 2-a-b \end{pmatrix} \succeq 0 \right\}$$

Facet-Ray Identification Property

Definition

Let G be a graph. For each facet F of $\mathrm{cut}^{\pm 1}(\mathrm{G})$ let $\alpha^F \in \mathbb{R}^E$ denote the normal vector to the supporting hyperplane of F. We say that G has the **facet-ray identification property** (or FRIP) if for every facet F of $\mathrm{cut}^{\pm 1}(\mathrm{G})$ there exists an extremal matrix $M = [m_{ij}]$ in $\mathcal{K}_G$ for which either $m_{ij} = \alpha^F_{ij}$ for every $\{i, j\} \in E(G)$ or $m_{ij} = -\alpha^F_{ij}$ for every $\{i, j\} \in E(G)$.

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Theorem [Solus, Uhler, Y. 2015]

Let $p \geq 3$. The cycle C_p on p vertices has the facet-ray identification property. Moreover, the cut polytope $\mathrm{cut}^{\pm 1}(\mathrm{C_p})$ is the p-halfcube which has two types of facets, halfcubical and simplicial. If the extremal matrix M is identified by a halfcubical facet then it is rank 1, and if it is identified by a simplicial facet it is rank $p - 2$.

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Theorem [Solus, Uhler, Y. 2015]

Every series-parallel graph has the facet-ray identification property. Moreover, the rank of the extremal ray is given by the constant term of the supporting hyperplane of the facet.

4-cycle: an example

The facets supported by the hyperplanes $\pm x_1 = 1$ correspond to the rank 1 extremal matrices

$$Y = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

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The facets $\langle v_T, x\rangle = 2$ for $v_T = (1, -1, 1, 1)$ and $v_T = (1, -1, -1, -1)$ respectively correspond to the rank 2 extremal matrices

$$Y = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \quad \text{and} \quad Y = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}.$$

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These four matrices respectively project to the four extreme points in $\mathcal{E}_G^\vee$

$$(1,0,0,0), \quad (-1,0,0,0), \quad \frac{1}{3}(-1,1,-1,-1), \quad \text{and} \quad \frac{1}{3}(-1,1,1,1),$$

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- R package
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Questions?