# GTM: Some topics in Commutative Algebra and Algebraic Geometry

Genova, February 24-25, 2011

### Grothendieck-Lefschetz Theory, Set-Theoretic Complete Intersections and Rational Normal Scrolls

Lucian Badescu

Using the Grothendieck-Lefschetz theory and a generalization (due to Cutkosky) of a result of Grothendieck (SGA2) concerning the simple connectedness, I will present a general result implying that many closed subvarieties of $\mathbf{P}^{\mathbf{N}}$ of dimension $\geq 2$ need at least $N-1$ equations to be defined in $\mathbf{P}^{\mathbf{N}}$ set-theoretically, i.e. their arithmetic rank is $\geq N-1$. As applications I will give a number of relevant examples. In the second part of the paper I will present a theorem computing the arithmetic rank of a rational normal scroll of dimension $d \geq 2$ in $\mathbf{P}^{\mathbf{N}}$ is $N-2$, by producing an explicit set of $N-2$ homogeneous equations which define these scrolls set-theoretically. This is joint work with Tito Valla.

### Su alcuni esempi di varieta' di Calabi-Yau

Gilberto Bini

In questo seminario verranno presentati alcuni esempi particolarmente interessanti di varieta' di Calabi-Yau. Una delle costruzioni si basa su un rivestimento dello spazio proiettivo complesso di dimensione tre con luogo di diramazione una superficie di grado otto ed e' frutto di una collaborazione con Bert van Geemen. Un'altra costruzione si basa sui quozienti di varieta' immerse come sezioni dell'anticanonico di una Fano fourfold data dal prodotto di due superfici di del Pezzo. Si tratta di una collaborazione con Filippo F. Favale.

### Prym map and 2nd gaussian maps.

Paola Frediani

I will report on some very recent results obtained in collaboration with Elisabetta Colombo on the rank of the second gaussian map for Prym-canonical line bundles on a general curve and its relation with the second fundamental form of the Prym map.

### Wronski–Schubert Calculus

Letterio Gatto

I will report on a joint work with Inna Scherbak (University of Tel Aviv). The *generalized wronskians* are natural and useful extensions of the classical notion of the wronskian of a $n$-tuple of differentiable functions. They appeared already in 1939, used by Schmidt (Math. Z., **45**) for applications to Weierstrass points on curves and investigated more recently, with similar motivations, by Ponza (Tesi di Dottorato, C.U. Torino – Genova, 1997) and Towse (Pacific J. Math., **193**, 2, 2000). For their applications to number theory see Anderson (J. Number Th., **115**, 2, 2005) and Milas – Mortenson – Onoper (Int. J. Number Theory, **4**, 2008).

The goal of the talk is to prove that generalized wronskians associated to a fundamental system of solutions of a linear ordinary differential equation with constant coefficients enjoy a *Giambelli's like formula*. More precisely, the ratio between a generalized wronskian and a wronskian is a Schur polynomial in certain polynomial expressions of the coefficients of the given differential equation. This generalizes the well known theorem due to Abel and Liouville claiming that the wronskian and its first derivative are proportional.

The relationship of such a result with the intersection theory on Grassmann varieties, ruled by *Schubert Calculus*, will be discussed. Schubert cycles can be seen as wronskians associated to a fundamental system of solutions of a linear ODE with coefficients taken in the intersection ring of the grassmannian. To say it with a slogan, "Schubert Calculus is Wronski Calculus" (and conversely).

**Borel-fixed ideals and Hilbert schemes**

Paolo Lella

In the study of Hilbert schemes, points corresponding to subschemes defined by Borel-fixed ideals play a special role, indeed by a Galligo's theorem we know that every component and every intersection of components of the Hilbert scheme contain at least one of such points. Moreover if we consider a polynomial ring with coefficients in an algebraically closed field of characteristic 0, Borel ideals are characterized by a strong combinatorial property that provides interesting ideas for a computational study of Hilbert schemes.

In the first part of the talk, we will discuss the combinatorial properties of such ideals and we will exhibit an algorithm computing all Borel-fixed ideals with assigned number of variables and Hilbert polynomial. Afterwards we will introduce a technique of determining deformations between couples of Borel ideals and we will prove that such deformations correspond to rational curves on the Hilbert scheme. Starting from these deformations, it is possible to give a new proof of the connectedness of the Hilbert scheme.

**Stable Complete Intersections**

(joint work with Maria Laura Torrente)

L. Robbiano - J. Abbott

Whatever algorithm is used to compute the defining ideal of a finite set of points, at a certain moment one has computed n polynomials $f_1, ..., f_n$ which generate a 0-dimensional complete intersection. Question: how do the zeros change when the coefficients of the polynomials $f_1, ..., f_n$ are perturbed? In the talk I will explain the motivation and the difficulties of tackling this problem. We face a mixture of classical algebro-geometric problems combined with numerical algebra methods. As a fundamental support for this research we need powerful computational tools which are provided by the new CoCoA5 and which will be presented by John Abbott during the talk.

**On the problem of connectedness for the Hilbert schemes of locally Cohen-Macaulay space curves**

E. Schlesinger

I will review what is known and what is not known about the connectedness of the Hilbert schemes of locally Cohen-Macaulay space curves.

**Irreducibility criterion for algebroid curves**

Takafumi Shibuta

The purpose of this talk is to give a criterion for algebroid curves (one-dimensional complete local rings) to be irreducible. To do this, we introduce a new notion of local tropical variety which is a straightforward extension of tropism introduced by Maurer.