Differential Equations in the Spectral Parameter, Soliton Equations, and Huygens' Principle

J. P. Zubelli
IMPA - Rio de Janeiro

This talk is concerned with seemingly unrelated subjects whose common thread surfaces from the theory of integrable nonlinear evolution equations.

The first one is a problem posed and studied by Duistermaat and Grünbaum. It was originally motivated by spectral theory of integral operators associated to signal processing and computerized tomography. It goes as follows: Characterize the differential operators $L(x, \partial_x)$ that admit a family of eigenfunctions $\varphi(x, \lambda)$ also satisfying a differential equation in the spectral parameter of the form

$$B(\lambda, \partial_\lambda)\varphi = \Theta(x)\varphi \ ,$$

where Θ is a function independent of λ.

The second subject is the theory of rational solutions of the KdV equation and its generalizations such as the Kadomtsev-Petviashvili equation. The rational solutions of KdV are limiting cases of the soliton solutions and play an important role in the theory of Calogero-Moser systems.

The third topic is a property of the domain of dependence of the wave equation solutions called Huygens' principle (in Hadamard's form). It is well known that if the number of space dimensions is odd and greater than 1, then the solution to the standard Cauchy problem for wave equation depends only on the values of the initial data along the intersection of the light cone with the initial condition manifold $t = 0$. A fairly natural question would be: Under what perturbations of the wave equation such a property is preserved? This is related with an important conjecture of Hadamard's and was extensively studied by Lagnese and collaborators during the seventies. In the early ninities it was revisited by Berest who made several progresses. In particular he called the attention to the connection of symmetries that preserve Huygens property with some results of Magri and Zubelli concerning the bispectral problem mentioned above.

In this talk we shall explain how the three topics are connected and report on some recent developments.