Dipartimento di Matematica
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A friendly chat about my research activity

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Solar Flares

Solar Flares

Main Interests

- Models for particle energy loss
- Description of particle motion
- Models for particle acceleration

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- Models for particle energy loss
- Description of particle motion
- Models for particle acceleration

Hot target model

Assumptions

- thermal background
- thick-target model: Coulomb collisions
- discard contributions due to Bremsstrahlung

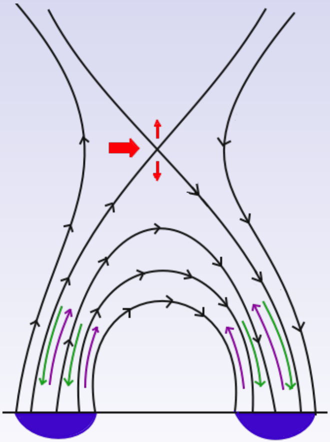
Hot target model

Background distribution : Maxwellian distribution

Energy loss rate : $\frac{dE}{ds} = -\frac{Kn\sqrt{\pi}}{2v^2}\left[\mathrm{erf}\left(\frac{v}{w_T}\right) - \frac{4}{\sqrt{\pi}}e^{-\frac{v^2}{w_T^2}}\right]$

Motivations for return currents

- emitted photons $\gg$ electrons in a typical coronal volume
- need of an injection term
- charge unbalance
- return currents restore charge equilibrium



Return currents models: ohmic losses

Assumptions

- two separate contributions
- Collisional contribution : hot target model
- Ohmic contribution : return currents driven by electric field

Ohmic losses

Background distribution : Maxwellian distribution
Energy loss rate :

$$\frac{dE}{ds} = \left(\frac{dE}{ds}\right)_{coll} + \left(\frac{dE}{ds}\right)_{ohm} = -\frac{Kn\sqrt{\pi}}{2v^2}\left[\text{erf}\left(\frac{v}{w_T}\right) - \frac{4}{\sqrt{\pi}}e^{-\frac{v^2}{w_T^2}}\right] - \frac{n\,e^2\,\eta v_0}{v^2}$$

Return currents models: Codispoti et al. 2013

Assumption

- return currents :coherent motion of background particles
- changes of collisional energy loss rate

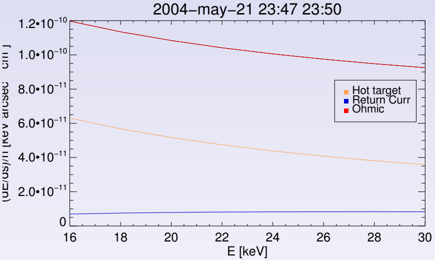
Codispoti et al. 2013

Background distribution : Maxwellian distribution shifted of a factor v_0
Collisional energy loss rate :

$$\left(\frac{dE}{ds}\right)_{coll} = -\frac{Kn\sqrt{\pi}}{2w_T v}\left[\left(\frac{w_T}{v+v_0} - 2\frac{w_T v_0}{|v+v_0|^2}\right)\operatorname{erf}\left(\frac{v+v_0}{w_T}\right) - \frac{4}{\sqrt{\pi}}e^{-\frac{(v+v_0)^2}{w_T^2}}\left(1-\frac{v_0}{v+v_0}\right)\right]$$

Comparision between models

- Codispoti et. al: lower profile and slower decrease



Models validation: Torre et al. 2012 method

Continuity equation: a different form

$$\frac{dE}{ds} - \frac{1}{F(s;E)} \int_E^{\infty} S(s;E)\, dE = \frac{1}{F(s;E)} \int_E^{\infty} \frac{\partial F(s;E)}{\partial s}\, dE$$

Theoretical models VS **Experimental data**

Theoretical models

- **Energy loss rate :**
  - Hot target model
  - Ohmic loss model
  - Codispoti et al. 2013 model
- **Source term :**
  - Extended source (Guo et al. 2012a): $\int_E^\infty S(s;E)dE = \begin{cases} \frac{h_0\, E_C^\delta}{(\delta-1)\, E^{\delta-1}} & |s| \leq \frac{L_0}{2} \\ 0 & |s| > \frac{L_0}{2} \end{cases}$

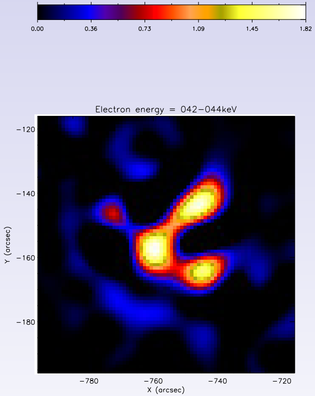
Experimental data

Electron flux : $F(s;E)$

- deduced using photon visibilities inversion algorithm (Piana et al. 2007).

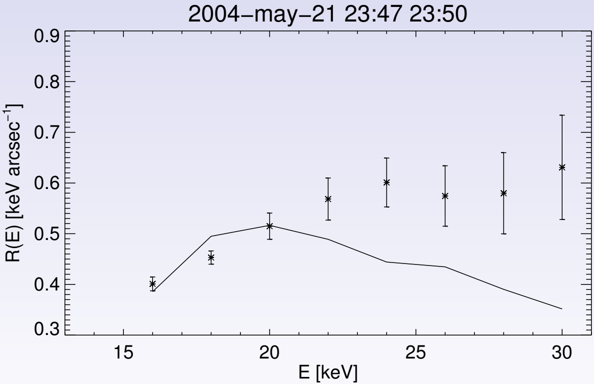
Results: May, 21st 2004 event

- **Time** $[UT]$ 23:47:00-23:50:00
- **Position** $[arcsec]$
  $[-756W, -156N]$
- **Class** M-flare
- **EM** $[10^{49} cm^{-3}]$
  0.354 ± 0.027



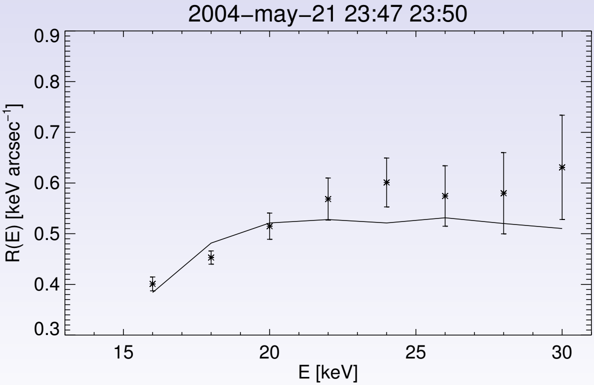
Hot target model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
0	$(2.23 \pm 0.14) \times 10^{10}$	$(1.16 \pm 0.11) \times 10^{31}$	40.91



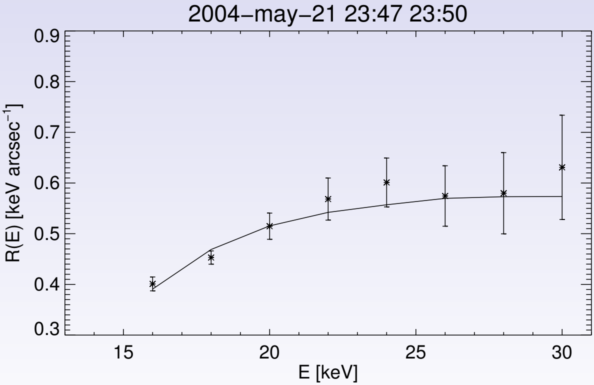
Ohmic loss model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
2.2 ± 0.3	$(9.12 \pm 0.09) \times 10^9$	$(7.3 \pm 0.6) \times 10^{30}$	13.56



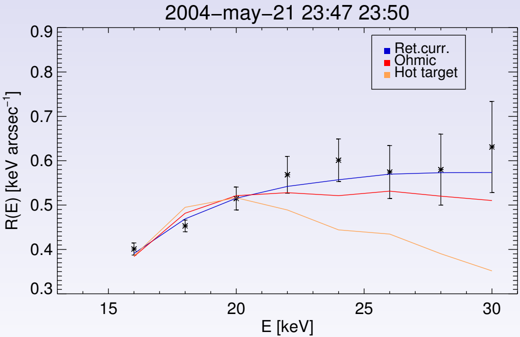
Codispoti et al. 2013 model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
8.7 ± 0.2	$(7.6 \pm 0.6) \times 10^{10}$	$(1.55 \pm 0.23) \times 10^{30}$	3.85



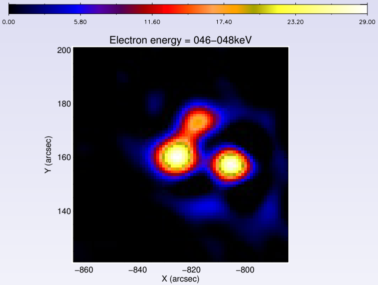
Models comparison

$EM[10^{49}cm^{-3}]$	Hot target	Ohmic losses	Drift velocity
0.62 ± 0.04	0.028 ± 0.009	0.0047 ± 0.0008	0.31 ± 0.11



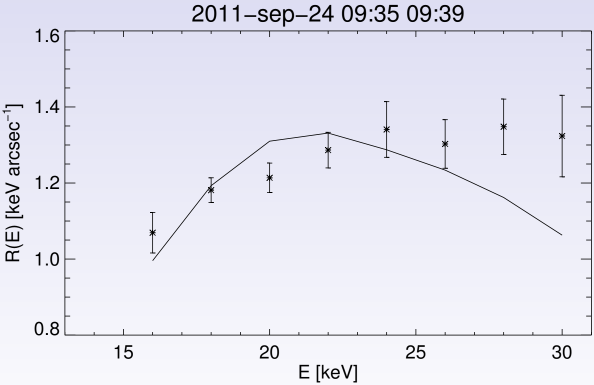
Results: September, 24th 2011 event

- **Time** $[UT]$ 09:35:00-09:39:00
- **Position** $[arcsec]$ $[-824W, 161N]$
- **Class** X-flare
- **EM** $[10^{49} cm^{-3}]$ 12.77 ± 1.99



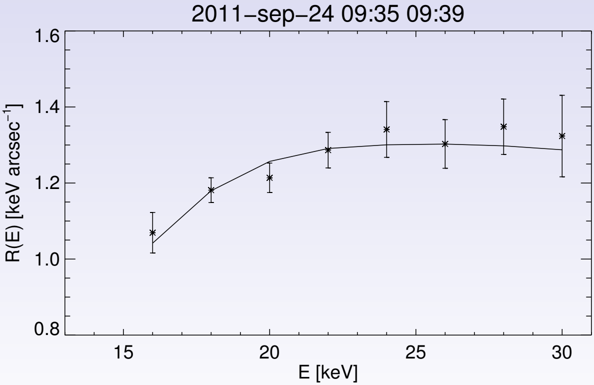
Hot target model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
0	$(3.7 \pm 0.1) \times 10^{10}$	$(4.4 \pm 0.3) \times 10^{32}$	21.44



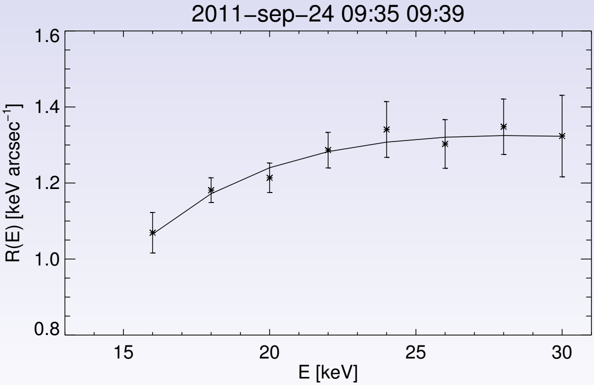
Ohmic loss model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
14.03 ± 2.95	$(8.2 \pm 0.5) \times 10^9$	$(1.8 \pm 0.2) \times 10^{32}$	2.42



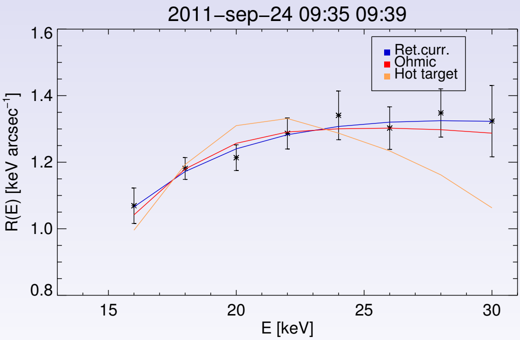
Codispoti et al. 2013 model

$E_0[keV]$	$n[cm^{-3}]$	$h_0[cm^{-5}keV^{-1}s^{-1}]$	χ^2
8.9 ± 0.5	$(1.6 \pm 0.1) \times 10^{11}$	$(8.06 \pm 1.23) \times 10^{30}$	0.95



Comparison of models

$EM[10^{49} cm^{-3}]$	Hot target	Ohmic losses	Drift velocity
12.77 ± 1.99	1.25 ± 0.17	0.04 ± 0.01	14.93 ± 3.15



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Motivations

Main goal

Description of the energy transport of accelerated electrons in the flaring region

Classic continuity equation presuming flux conservation

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f) + \nabla_{\mathbf{v}} \cdot \left(\frac{\mathbf{\Phi}}{m} f \right) + \nabla_{\mathbf{v}} \cdot \left(\frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} f \right) = S \quad (1)$$

Starting from the Boltzmann equation excluding the flux conservation assumption

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} f + \frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} \cdot \nabla_{\mathbf{v}} f = S \quad (2)$$

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The Boltzmann equation and the Landau approximation

The Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} f = Q(f, f) + S \qquad (3)$$

Boltzmann collision kernel

$$Q(f, f)(t, \mathbf{x}, \mathbf{v}) = \int_{\mathbb{R}^3} \int_{S^2} B(|\mathbf{v} - \mathbf{v_1}|, \sigma)(f' f_1' - f\, f_1) d^2\sigma d^3\mathbf{v_1} \qquad (4)$$

Rutherford cross section

- In the case of Coulomb collisions

$$B(|\mathbf{v} - \mathbf{v}_1|, \sigma) = \frac{e^4}{m^2} \frac{1}{|\mathbf{v} - \mathbf{v}_1|^3 \sin^4 \frac{\theta}{2}} \qquad \theta \geq \theta_0 \qquad (5)$$

- $\theta_0 \approx 4.12 \times 10^{-9}$ in solar corona

The Boltzmann equation and the Landau approximation

Small scattering angles: The Landau approximation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} f = Q_L(f, f) + S \tag{6}$$

Landau collision kernel

$$Q_L(f,f)(t,\mathbf{x},\mathbf{v}) = \frac{8\pi\Lambda e^4}{m^2} \nabla_{\mathbf{v}} \cdot \int_{\mathbb{R}^3} A(f_1 \nabla_{\mathbf{v}} f - f \nabla_{\mathbf{v_1}} f_1) d^3\mathbf{v}_1$$

$$a_{ij} = \frac{1}{4|\mathbf{v} - \mathbf{v}_1|} \left(\delta_{ij} - \frac{(\mathbf{v} - \mathbf{v}_1)_i (\mathbf{v} - \mathbf{v}_1)_j}{|\mathbf{v} - \mathbf{v}_1|^2} \right) \tag{7}$$

Linearization and asymptotic expansion of the Landau equation

$f(t,\mathbf{x},\mathbf{v})$ solution rapidly decreasing over $|\mathbf{v}|$, δf small perturbation

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\delta f + \frac{\mathbf{\Phi}}{m}\cdot\nabla_{\mathbf{v}}\delta f = (L_B + S_B)(\delta f)(t,\mathbf{x},\mathbf{v}) + S'(t,\mathbf{x},\mathbf{v})$$

$$L_B(\delta f)(t,\mathbf{x},\mathbf{v}) = \frac{8\pi\Lambda e^4}{m^2}\nabla_{\mathbf{v}}\cdot\int_{\mathbb{R}^3} A(f_1\nabla_{\mathbf{v}}\delta f - \delta f\nabla_{\mathbf{v_1}} f_1)d^3\mathbf{v}_1$$

$$S_B(\delta f)(t,\mathbf{x},\mathbf{v}) = \frac{8\pi\Lambda e^4}{m^2}\nabla_{\mathbf{v}}\cdot\int_{\mathbb{R}^3} A(\delta f_1\nabla_{\mathbf{v}} f - f\nabla_{\mathbf{v_1}}\delta f_1)d^3\mathbf{v}_1$$

Features of L_B and S_B

- S_B in integral form on δf, rapid decrease
- L_B acts locally in $\mathbf{v}$ on δf, chatacterizes asymptotic behavior of Q_L

Asymptotic expansion of the Landau kernel

Proposition

The **linearized Landau integral kernel** $Q_L = L_B + S_B$ *computed over a background distribution* $f(t, \mathbf{x}, \mathbf{v})$ *rapidly decreasing over* $|\mathbf{v}|$, *is such that*

$$Q_L(\delta f) = -\frac{8\pi\Lambda e^4}{m^2}\left(-\frac{n}{4|\mathbf{v}|}\Delta\delta f + \frac{n}{4}\frac{\mathbf{v}^i\mathbf{v}^j}{|\mathbf{v}|^3}\partial_{ij}\delta f\right) + R, \tag{8}$$

where R *is function which vanishes faster than* $1/|\mathbf{v}|$ *for large values of* $|\mathbf{v}|$.
In particular, in the case of δf **isotropic function**,*i.e.* $\delta f(\mathbf{v}) = g(|\mathbf{v}|)$,

$$Q_L(\delta f)(t, \mathbf{x}, \mathbf{v}) = \frac{8\pi\Lambda e^4}{m^2}\frac{n}{2|\mathbf{v}|^3}\mathbf{v}\cdot\nabla_{\mathbf{v}}\delta f + R. \tag{9}$$

Theorem

The continuity equation: some comments

Classic continuity equation (Emslie, Barrett and Brown 2001)

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f) + \nabla_{\mathbf{v}} \cdot \left(\frac{\mathbf{\Phi}}{m} f \right) + \nabla_{\mathbf{v}} \cdot \left(\frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} f \right) = S \quad (10)$$

Asymptotic incompatibility between Landau equation and continuity equation loss terms

Different formulation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} f + \frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} \cdot \nabla_{\mathbf{v}} f = S \quad (11)$$

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Effects on the continuity equation

Modified continuity equation

$$\frac{\partial \delta f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \delta f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} \delta f = -\frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} \cdot \nabla_{\mathbf{v}} \delta f + S \tag{12}$$

$$Q_C(\delta f)(t, \mathbf{x}, \mathbf{v}) = -\frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} \cdot \nabla_{\mathbf{v}} \delta f \tag{13}$$

Proposition

Let $f(t, \mathbf{x}, \mathbf{v})$ be the background distribution of the plasma particles. Assuming f is a function rapidly decreasing over $|\mathbf{v}|$, the asymptotic expansion of **the continuity equation loss term** $Q_C(t, \mathbf{x}, \mathbf{v})$ *for large $|\mathbf{v}|$ is such that*

$$Q_C(t, \mathbf{x}, \mathbf{v}) = \frac{8\pi \Lambda e^4}{m^2} \frac{n}{2|\mathbf{v}|^3} \mathbf{v} \cdot \nabla_{\mathbf{v}} \delta f + R \tag{14}$$

where, for large values of $|\mathbf{v}|$, R is a function that vanishes faster than $1/|\mathbf{v}|^3$.

Theorem

Resolution of the continuity equation

Modified continuity equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{\Phi}}{m} \cdot \nabla_{\mathbf{v}} f = -\frac{\langle \nabla_{\mathbf{x}} E \rangle}{m} \cdot \nabla_{\mathbf{v}} f + S \quad (15)$$

Measured data are related to the electron flux $F(t, \mathrm{x}, E)$: no information about velocities directions

We want to write it for the electron flux $F(t, \mathrm{x}, E)$

$$\int_{\mathbb{S}^2} f(t, \mathbf{x}, \mathbf{v}) d\mathbb{S}^2 = m^2 \frac{F(t, \mathbf{x}; E)}{2E} \quad (16)$$

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Resolution of the continuity equation

Assumptions

- Motion along magnetic field lines
- Evolution of f does not vary much passing to nearby magnetic field lines
- f and S depend only by $|\mathbf{v}|$
- $f = f_+ + f_-$ and $S = S_+ = S_-$

Integrating over $d\mathbb{S}^2$, looking for stationary solutions in the absence of external forces

$$\pm\frac{\partial}{\partial s}\frac{F_\pm(s,E)}{E} + 2\left\langle\frac{dE}{ds}\right\rangle\frac{\partial}{\partial E}\frac{F_\pm(s,E)}{E} = \sqrt{\frac{2m}{E}}\frac{\Sigma(s,E)}{E} \quad (17)$$

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Resolution of the continuity equation

Solution

$$F(s,E) = 2E \int \sqrt{\frac{2m}{E(x+|s-s_0|)}} \frac{\Sigma(s_0, E(x+|s-s_0|))}{E(x+|s-s_0|)} ds_0 \quad (18)$$

$$x(E) = x_0 + \int_{E_0}^{E} \frac{1}{2\left\langle \frac{dE}{ds} \right\rangle} dE'$$

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- Models for particle acceleration

Motivations

State of the art

- Xu et al. 2008: need of an extended region for the particle acceleration
- Guo et al. 2013: analysis of spatially integrated quantities in order to compare different models for particle acceleration $\rightarrow$ no substantial differences

Our purpose

Analysis of spatially resolved electron flux $F(s, E)$ in order to select the best model for particle acceleration

Main instrument

Solution for the continuity equation

$$F(s,E) = 2E \int \sqrt{\frac{2m}{E(x+|s-s_0|)}} \frac{\Sigma(s_0, E(x+|s-s_0|))}{E(x+|s-s_0|)} ds_0 \tag{19}$$

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Main instrument

Solution for the continuity equation

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$$x(E) = x_0 + \int_{E_0}^{E} \frac{1}{2\left\langle \frac{dE}{ds} \right\rangle} dE'$$

Models for the analysis

Energy loss rate: Hot target model

$$\left\langle \frac{dE}{ds} \right\rangle = -\frac{Kn}{E}\left[\mathrm{erf}\left(\sqrt{\frac{E}{kT}}\right) - \frac{4}{\sqrt{\pi}}\sqrt{\frac{E}{kT}}\, e^{-\frac{E}{kT}}\right]$$

Acceleration Models

- Box model:

$$\Sigma(s,E) = h_s \sqrt{\frac{E}{2m}} \left(\frac{E}{E_0}\right)^{-\delta} \Theta\left(\frac{L_0}{2} - |s - s_m|\right),$$

- Gaussian model:

$$\Sigma(s,E) = h_s \sqrt{\frac{E}{2m}} \left(\frac{E}{E_0}\right)^{-\delta} e^{\frac{(s-s_m)^2}{2\sigma^2}}$$

Method for the analysis

Method

Nelder-Mead method with MSEP as energy function

Data

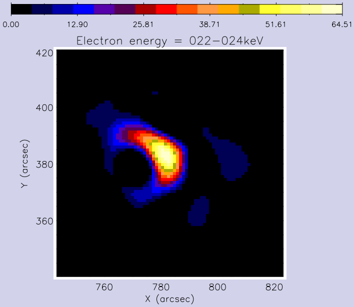
- Number of analyzed events: 18
- Electron energy range: 20-34 keV
- Empirical electron flux: RHESSI DATA + Piana et al. 2007 inversion algorithm
- Arclength selection: Torre et al. 2012

Results

- The Gaussian shaped model **systematically** provides the lower value for the MSEP
- Both models provide fitted parameters which are **reasonably** in accordance with parameters provided by other observation
- Sometimes the Box shaped model is clearly **too rough** for the description of the acceleration mechanism

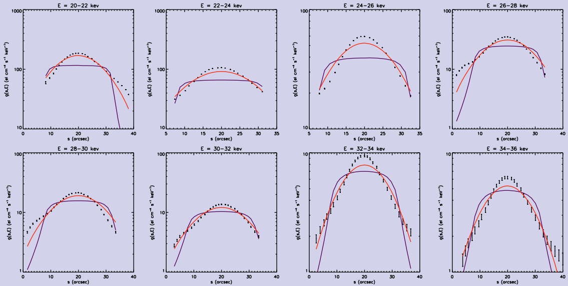
Just an example... 2002 Apr 15 event

15th April 2002 00:00-00:05



Just an example... 2002 Apr 15 event

Fit results



Resulting parameters

	BOX	GAUSS
h_0	$(6.366 \pm 0.552) \times 10^{27}$	$(1.118 \pm 0.115) \times 10^{28}$
L_0	24.761 ± 1.054	20.689 ± 1.038
δ_0	7.817 ± 0.121	8.313 ± 0.123
MSEP	0.103	0.023

Thank you for your attention!

Asymptotic expansion of the Landau kernel for $f \in \mathcal{S}(\mathbb{R}^3)$

L_B asymptotic expansion

$$L_B(\delta f) = -C\left[-2\pi\delta f f - \frac{\Delta\delta f}{4}\int_{\mathbb{R}^3}\frac{f_1}{|\mathbf{v}-\mathbf{v}_1|}d^3v_1 + \frac{\partial_{ij}\delta f}{4}\int_{\mathbb{R}^3}\frac{(\mathbf{v}-\mathbf{v}_1)^i(\mathbf{v}-\mathbf{v}_1)^j}{|\mathbf{v}-\mathbf{v}_1|^3}f_1 d^3v_1\right].$$

Theorem

Let $f(t,\mathbf{x},\mathbf{v})$ be a function rapidly decreasing over $|\mathbf{v}|$ and $n := \int_{\mathbb{R}^3} f_1 d^3v_1$. The following relations hold

1) $$\int_{\mathbb{R}^3}\frac{f_1}{|\mathbf{v}-\mathbf{v}_1|}d^3v_1 = \frac{n}{|\mathbf{v}|} + R_1(\mathbf{v}),$$

2) $$\int_{\mathbb{R}^3}\frac{(\mathbf{v}-\mathbf{v}_1)^i(\mathbf{v}-\mathbf{v}_1)^j}{|\mathbf{v}-\mathbf{v}_1|^3}f_1 d^3v_1 = \frac{\mathbf{v}^i\mathbf{v}^j}{|\mathbf{v}|^3}n + R_2(\mathbf{v}),$$

3) $$\int_{\mathbb{R}^3}\frac{\partial_i f_1}{|\mathbf{v}-\mathbf{v}_1|}d^3v_1 = \frac{\mathbf{v}^i}{|\mathbf{v}|^3}n + R_3(\mathbf{v}),$$

where R_1 and R_2 are functions that vanish faster than $1/|\mathbf{v}|$ for large $|\mathbf{v}|$, while R_3 is a function that vanishes faster than $1/|\mathbf{v}|^2$ for large values of $|\mathbf{v}|$.

Proof Continuity

Proof of Theorem

Lemma

Let f be a function in $\mathcal{S}(\mathbb{R}^2)$, $\mathbf{v} \in \mathbb{R}^3$. Consider the function $h(\mathbf{v}) := |\mathbf{v}|f(\mathbf{v})$, hence the Fourier transform of the Green operator applied to h, namely the Fourier transform of

$$\mathcal{G}(h) := \int_{\mathbb{R}^2} \frac{|\mathbf{v}_1| f_1}{|\mathbf{v} - \mathbf{v}_1|} d^3 v_1, \tag{21}$$

is a function in the L^1 space.

Proof of 1)

$$\lim_{|\mathbf{v}|\to\infty} |\mathbf{v}| \int_{\mathbb{R}^3} \frac{f_1}{|\mathbf{v}-\mathbf{v}_1|} d^3 v_1 = \lim_{|\mathbf{v}|\to\infty} \left[\int_{\mathbb{R}^3} \frac{(|\mathbf{v}|-|\mathbf{v}_1|)f_1}{|\mathbf{v}-\mathbf{v}_1|} d^3 v_1 + \int_{\mathbb{R}^3} \frac{|\mathbf{v}_1|f_1}{|\mathbf{v}-\mathbf{v}_1|} d^3 v_1 \right] = n$$

Back

Resolution of the continuity equation

Reduction to the wave equation

- $A = \sqrt{\frac{2m}{E}} \frac{\Sigma(s,E)}{E}$
- $\phi_\pm = \frac{F_\pm(s,E)}{E}$
- $x(E) = x_0 - \int_{E_0}^{E} \frac{1}{2\langle \nabla_{\mathbf{x}} E \rangle} dE'$

$$\pm \frac{\partial}{\partial s}\phi_\pm - \frac{\partial}{\partial x}\phi_\pm = A \quad (22)$$

Applying $\pm\frac{\partial}{\partial s} + \frac{\partial}{\partial x}$: Wave equation

$$-\frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2}{\partial s^2}\phi = 2\frac{\partial}{\partial x}A \quad (23)$$

where x time and s space

Resolution of the continuity equation

Boundary conditions

$$\lim_{x \to \infty} \phi(x, s \pm x) = 0, \qquad \lim_{x \to \infty} \partial_x \phi(x, s \pm x) = 0, \forall s \tag{24}$$

Fundamental solution

$\delta_- = \delta(x + |s|)$ solution of

$$-\frac{\partial^2}{\partial x^2}\delta_- + \frac{\partial^2}{\partial s^2}\delta_- = \partial_x \delta \tag{25}$$

Solution of the equation

$$\phi = \delta_- * 2A \tag{26}$$