Imaging concept and image reconstruction methods for the STIX instrument onboard the ESA Solar Orbiter mission

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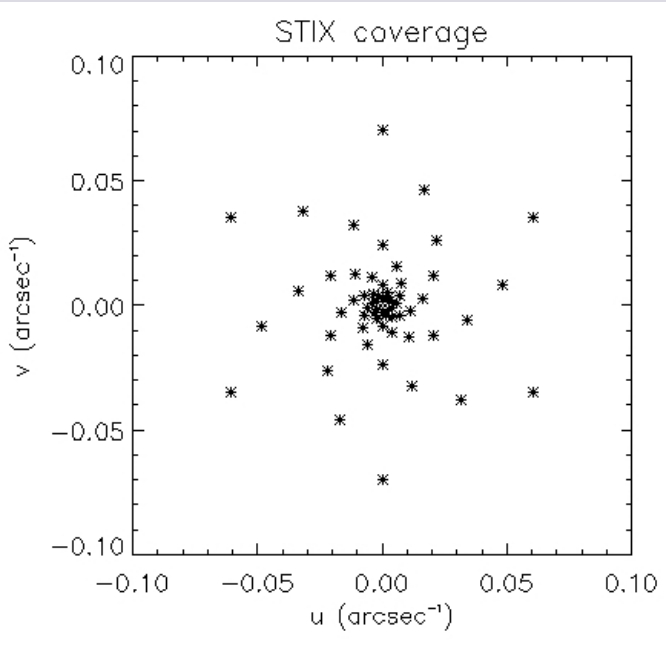
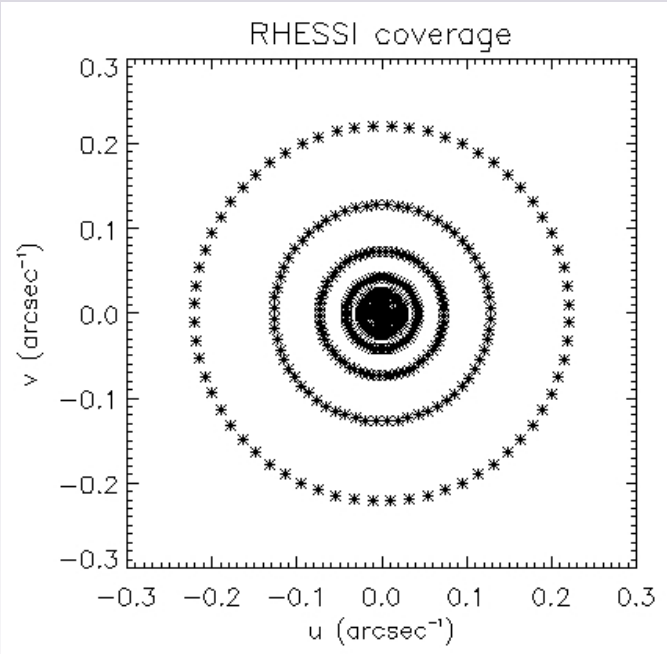
STIX - RHESSI data

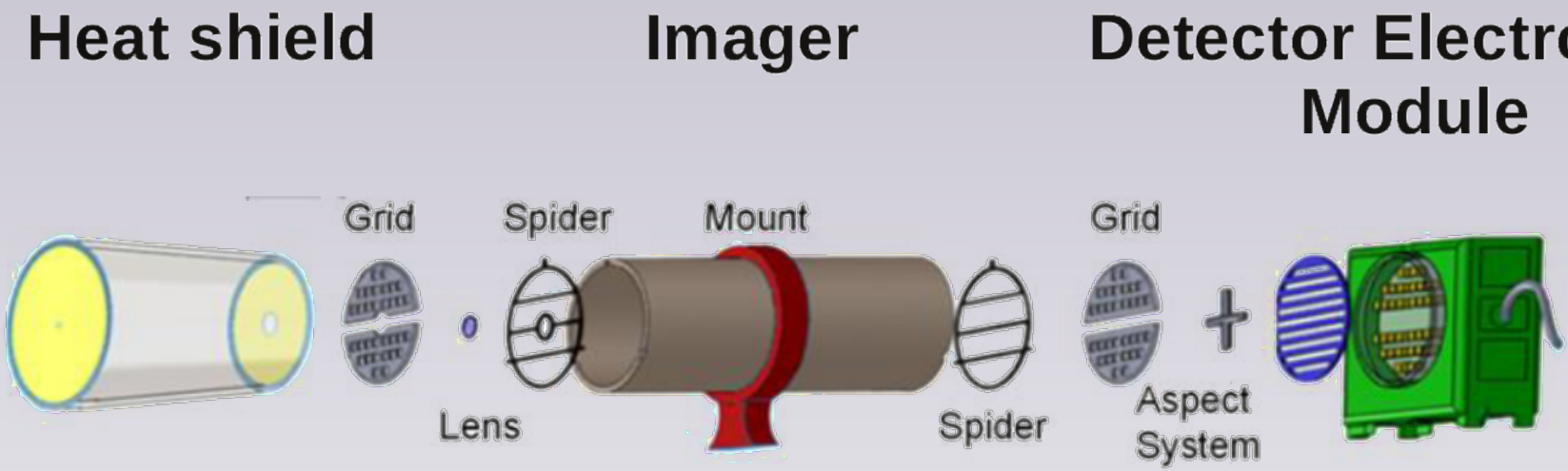
Visibility

$$V(u,v) = \iint I(x,y) e^{i2\pi(xu+yv)} dxdy$$

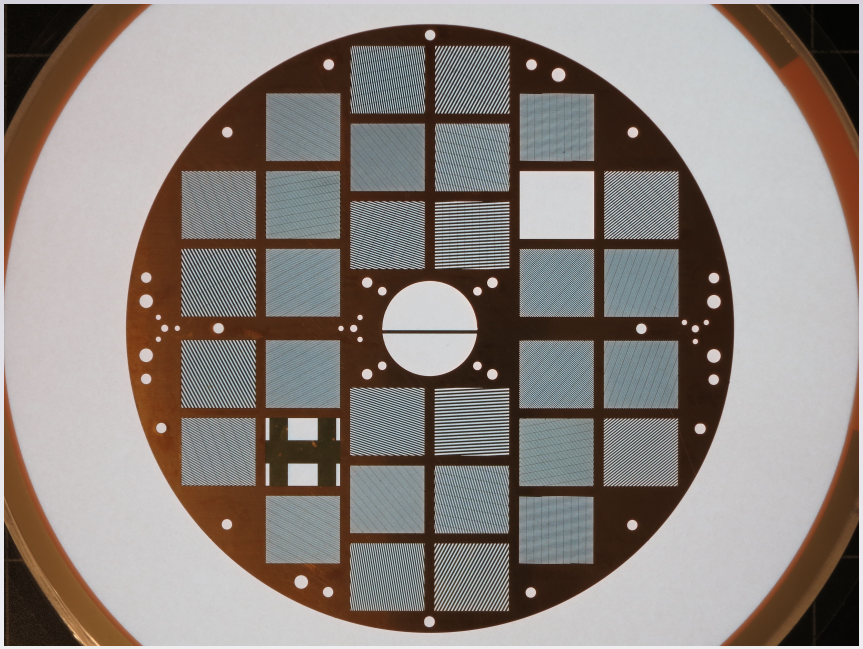


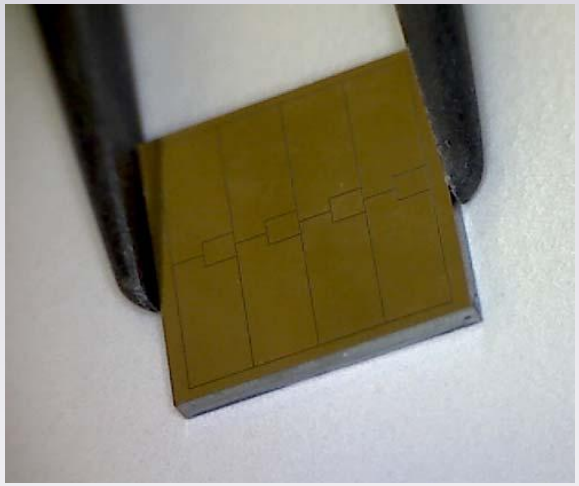


The STIX imaging concept

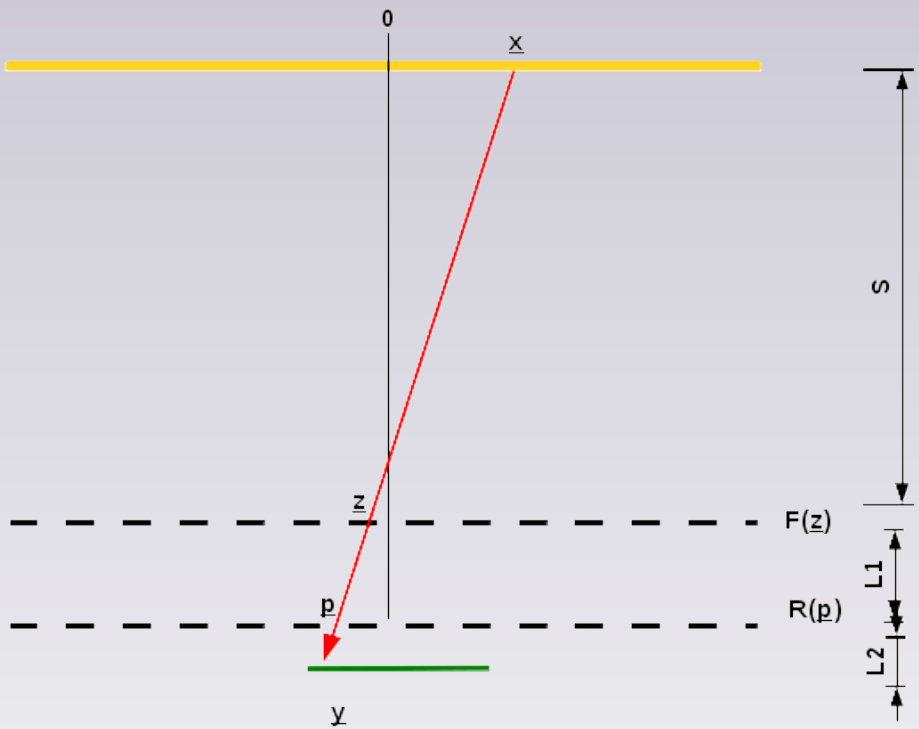


STIX grids





STIX data

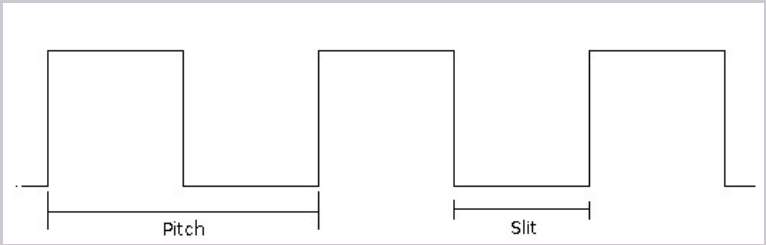


- $\phi(\mathbf{x})$ flux from the point **x**
- $F(\mathbf{z})$ transmission through the front grid
- $R(\mathbf{p})$ transmission through the rear grid
- $T(\mathbf{x}, \mathbf{y}) = F(\mathbf{z})R(\mathbf{p})$ transmission through the two grids
- P_n is a pixel of the detector $n = -2, -1, 0, 1$
- $\mathbf{k}^f = \left(\frac{\cos(\alpha_f)}{\text{pitch}_f}, \frac{\sin(\alpha_f)}{\text{pitch}_f} \right)$
- $\mathbf{k}^r = \left(\frac{\cos(\alpha_r)}{\text{pitch}_r}, \frac{\sin(\alpha_r)}{\text{pitch}_r} \right)$

Counts in each pixel

$$\text{Counts}_{P_n} = \int \int_{P_n} \phi(\mathbf{x}) T(\mathbf{x}, \mathbf{y}) d\mathbf{y} d\mathbf{x} = \int \phi(\mathbf{x}) \tau_n(\mathbf{x}) d\mathbf{x}$$

Section of a grid



Fourier series - front grid $F(\mathbf{x}, \mathbf{y})$

$$\frac{1}{2}\left[1+\sum_{m=1,3,5\ldots}^{\infty} b_m \sin\left(m2\pi\mathbf{k}^f \cdot \left(\mathbf{y}-\mathbf{x}\frac{L_1+L_2}{S}\right)\right)\right]$$

Fourier series - rear grid $R(\mathbf{x}, \mathbf{y})$

$$\frac{1}{2}\left[1+\sum_{m=1,3,5\ldots}^{\infty} b_m \sin\left(m2\pi\mathbf{k}^r \cdot \left(\mathbf{y}-\mathbf{x}\frac{L_2}{S}\right)\right)\right]$$

Transmission through the two grids

$$\begin{aligned} T(\mathbf{x},\mathbf{y}) = \frac{1}{4}\Bigg[1 &+ \sin\left(2\pi\mathbf{k}^f \cdot \left(\mathbf{y}-\mathbf{x}\frac{L_1+L_2}{S}\right)\right) + \sin\left(2\pi\mathbf{k}^r \cdot \left(\mathbf{y}-\mathbf{x}\frac{L_2}{S}\right)\right) \\ &- \frac{1}{2}\cos\left(-2\pi\left(\frac{L_1+L_2}{S}\mathbf{k}^f + \frac{L_2}{S}\mathbf{k}^r\right)\cdot\mathbf{x} + 2\pi\left(\mathbf{k}^r+\mathbf{k}^f\right)\cdot\mathbf{y}\right) \\ &+ \frac{1}{2}\cos\left(-2\pi\left(\frac{L_1+L_2}{S}\mathbf{k}^f - \frac{L_2}{S}\mathbf{k}^r\right)\cdot\mathbf{x} + 2\pi\left(\mathbf{k}^r-\mathbf{k}^f\right)\cdot\mathbf{y}\right)\Bigg] \end{aligned}$$

$$\mathbf{k}^{f/r} = \left(\frac{\cos(\alpha_{f/r})}{\text{pitch}_{f/r}}, \frac{\sin(\alpha_{f/r})}{\text{pitch}_{f/r}}\right)$$

Computation of counts in each pixel

Counts on P_n with $n = -2, -1, 0, 1$

$$\int \phi(\mathbf{x})\tau_n(\mathbf{x})d\mathbf{x} \simeq \frac{1}{4}\Bigg[M_0 V_\phi(\mathbf{0}) + M_1 \exp\left(i\frac{\pi}{2}n\right)\exp\left(i\frac{\pi}{4}\right)V_\phi(-\boldsymbol{\xi}) + M_1 \exp\left(-i\frac{\pi}{2}n\right)\exp\left(-i\frac{\pi}{4}\right)V_\phi(\boldsymbol{\xi})\Bigg]$$

$$V_\phi(\boldsymbol{\xi}) = \iint \phi(\mathbf{x})e^{i2\pi\boldsymbol{\xi}\cdot\mathbf{x}}d\mathbf{x} \qquad \boldsymbol{\xi} = \mathbf{k}^f\frac{L_1+L_2}{S} - \mathbf{k}^r\frac{L_2}{S}$$

$$\text{Counts}_{P_{-2}} = A = M_0F_0 - M_1\exp\left(i\frac{\pi}{4}\right)V_\phi(-\boldsymbol{\xi}) - M_1\exp\left(-i\frac{\pi}{4}\right)V_\phi(\boldsymbol{\xi})$$

$$\text{Counts}_{P_{-1}} = B = M_0F_0 - iM_1\exp\left(i\frac{\pi}{4}\right)V_\phi(-\boldsymbol{\xi}) + iM_1\exp\left(-i\frac{\pi}{4}\right)V_\phi(\boldsymbol{\xi})$$

$$\text{Counts}_{P_0} = C = M_0F_0 + M_1\exp\left(i\frac{\pi}{4}\right)V_\phi(-\boldsymbol{\xi}) + M_1\exp\left(-i\frac{\pi}{4}\right)V_\phi(\boldsymbol{\xi})$$

$$\text{Counts}_{P_1} = D = M_0F_0 + iM_1\exp\left(i\frac{\pi}{4}\right)V_\phi(-\boldsymbol{\xi}) - iM_1\exp\left(-i\frac{\pi}{4}\right)V_\phi(\boldsymbol{\xi})$$

Visibility computation

Real and imaginary parts of visibilities

$$C - A = 4M_1 \,\mathrm{Re}\left(\exp\left(-i\frac{\pi}{4}\right) V_\phi(\boldsymbol{\xi})\right)$$

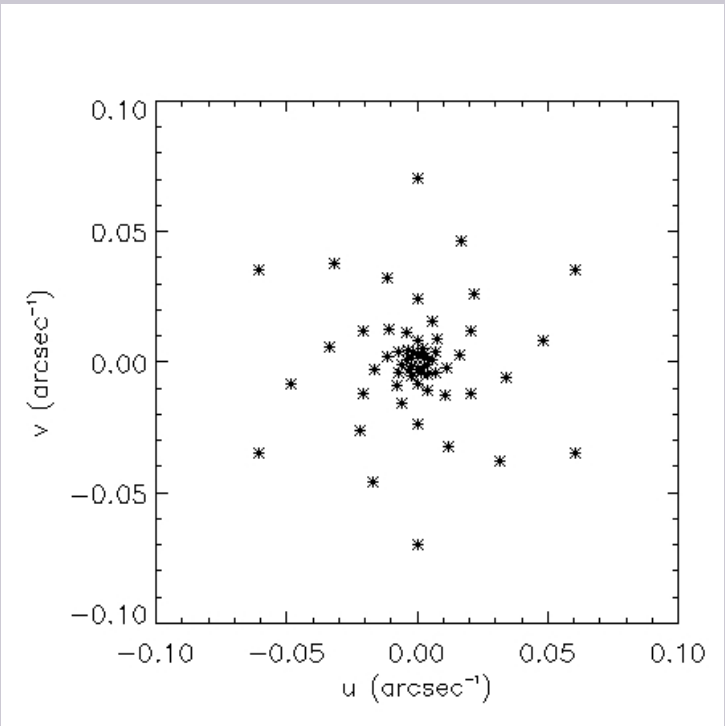
$$D - B = 4M_1 \,\mathrm{Im}\left(\exp\left(-i\frac{\pi}{4}\right) V_\phi(\boldsymbol{\xi})\right)$$

Visibilities

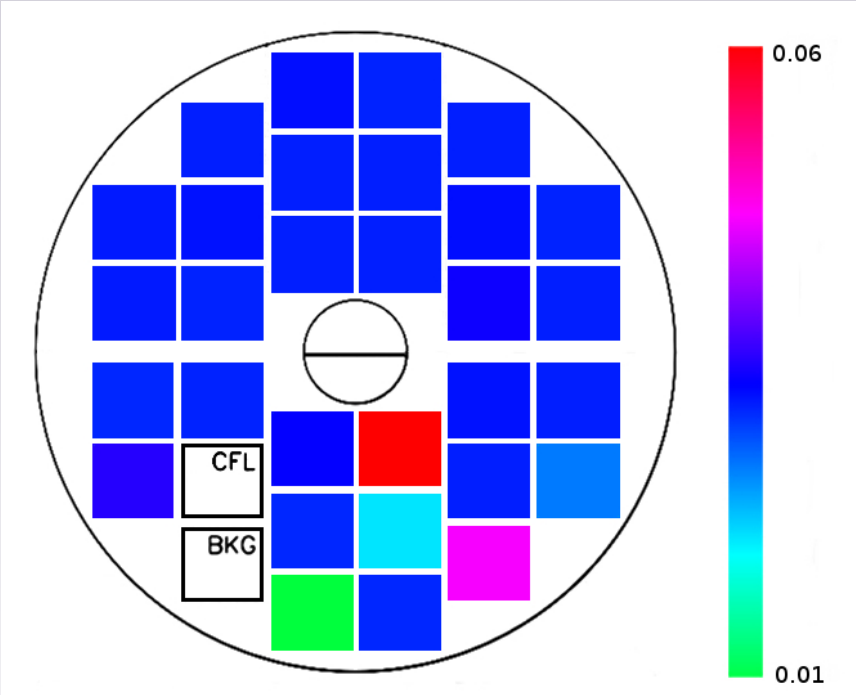
$$4M_1 V_\phi(\boldsymbol{\xi}) = ((C - A) + i(D - B))) \exp\left(i\frac{\pi}{4}\right)$$

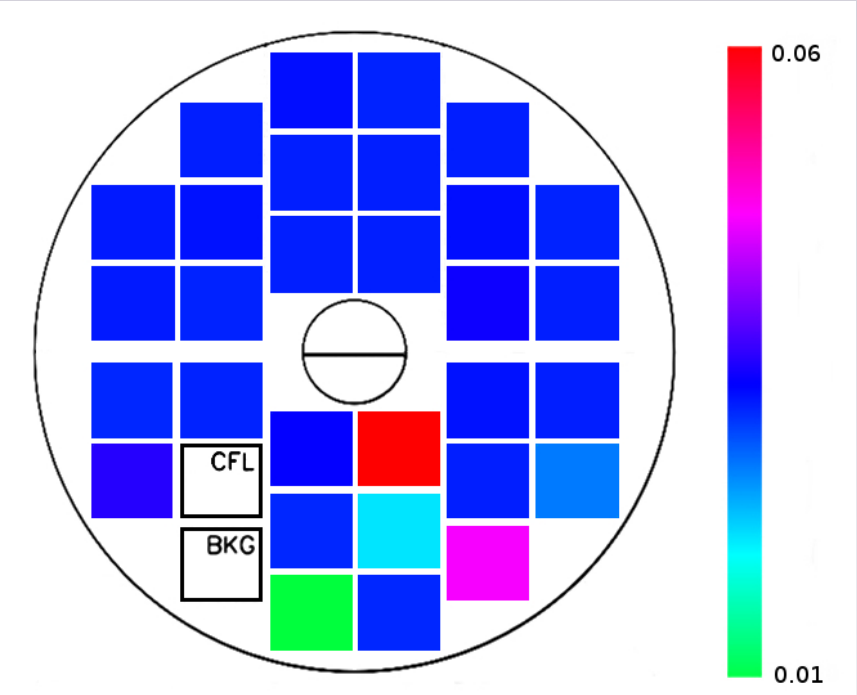
$$\boldsymbol{\xi} = \boldsymbol{\xi}(\alpha_r, \alpha_f, \mathsf{pitch}_r, \mathsf{pitch}_f)$$

uv - coverage



Approximation error





Deconvolution problem

$$I^D(x,y) = \iint I(x',y')K(x-x',y-y')dx'dy' = (I * K)(x,y) \tag{1}$$

where $I(x,y)$ is the source map , $K(x,y)$ is the PSF and $I^D(x,y)$ is the blurred map.

Solution model

$$I(x,y) = \sum_{q=1}^{N} I_q \delta(x-x_q, y-y_q) + B(x,y) \tag{2}$$

where I_q and (x_q, y_q) are respectively the amplitude and position of the q-th source and $B(x,y)$ is the background map.

$$I^D(x,y) = \sum_{q=1}^{N} I_q K(x-x_q, y-y_q) + (B * K)(x,y) \tag{3}$$

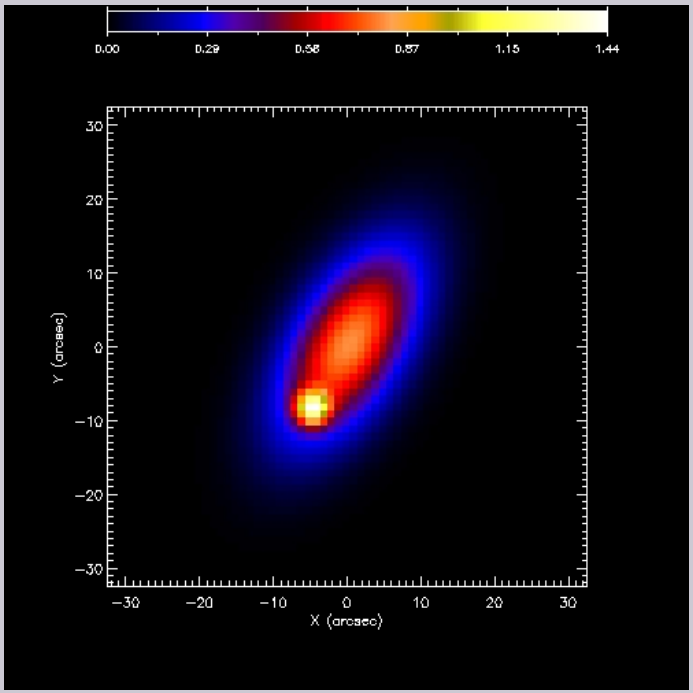
$I^D(x, y)$ Estimation

Back-projection

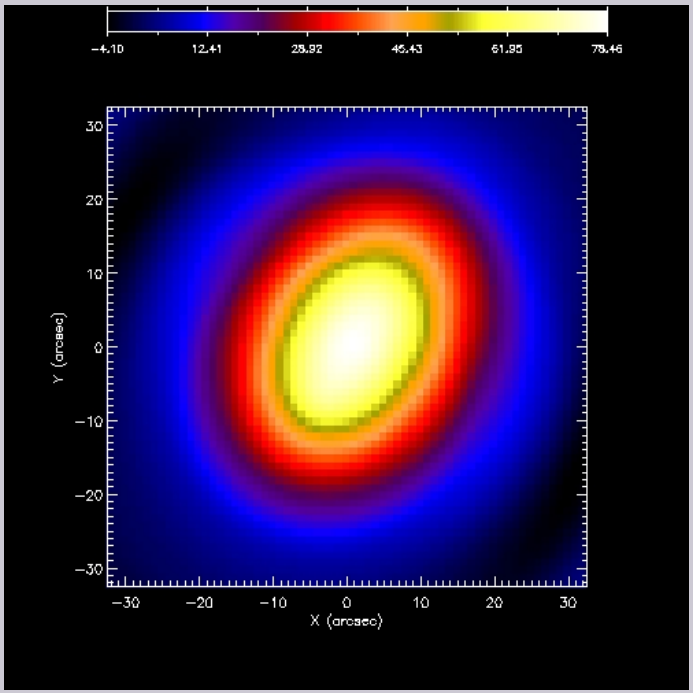
$$I(x, y) = \iint V(u, v) e^{-i2\pi(xu+yv)} du dv \tag{4}$$

$$I^D(x, y) = \sum_{j=0}^{N_{vis}} V(u_j, v_j) e^{-i2\pi(xu_j+yv_j)} \delta u \delta v \tag{5}$$

Simulated map



Reconstruction with back-projection

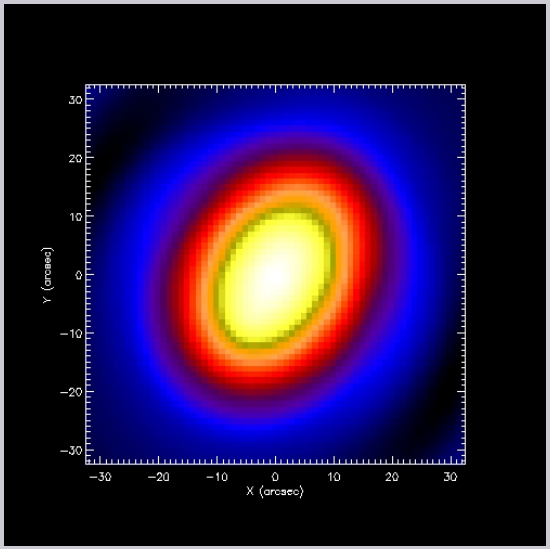


CLEAN loop (t-th iteration)

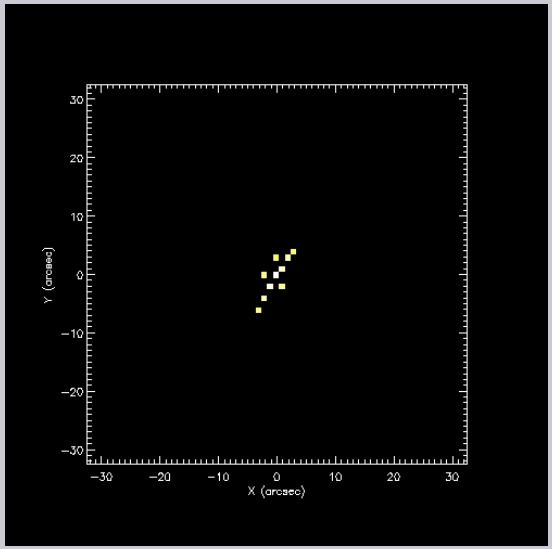
- $(x_t, y_t) = \arg\max_{(x,y)} I^D(x, y)$
- $CC(x, y) \leftarrow CC(x, y) + F_t \cdot \delta(x - x_t, y - y_t)$
- $I^D(x, y) \leftarrow I^D(x, y) - F_t \cdot K(x - x_t, y - y_t)$

STOP when there are no more sources

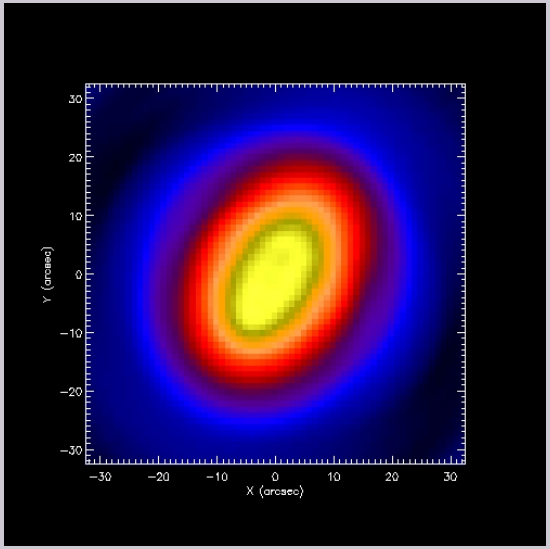
Dirty map



CC map



$I^D(x, y)$ updated

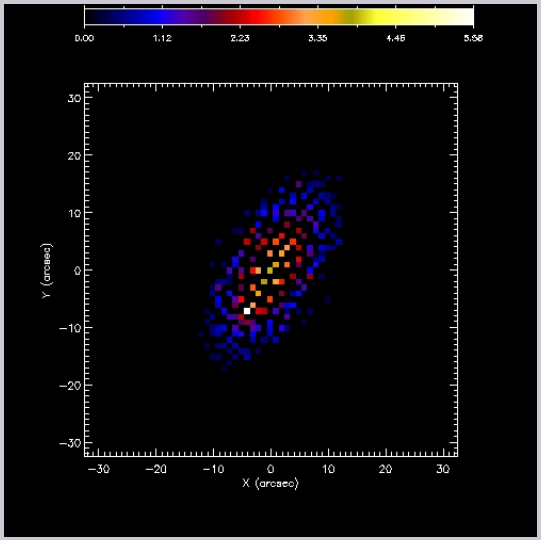


CLEAN loop output

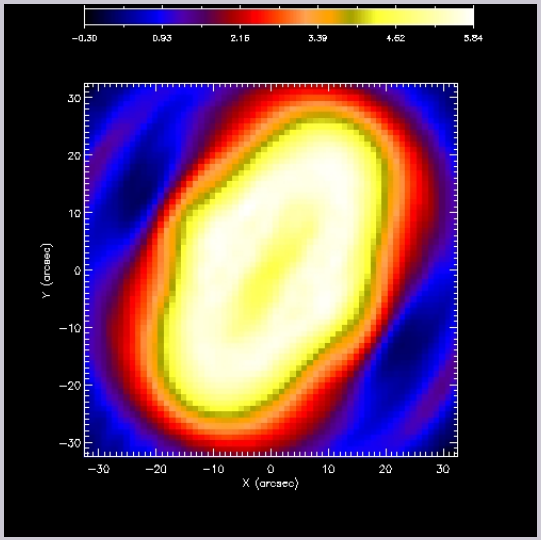
$$CC(x,y) = \sum_{t=1}^{N_{iter}} F_t \delta(x - x_t, y - y_t)$$

$$R(x,y) \simeq (B * K)(x,y)$$

CC map



Residual map

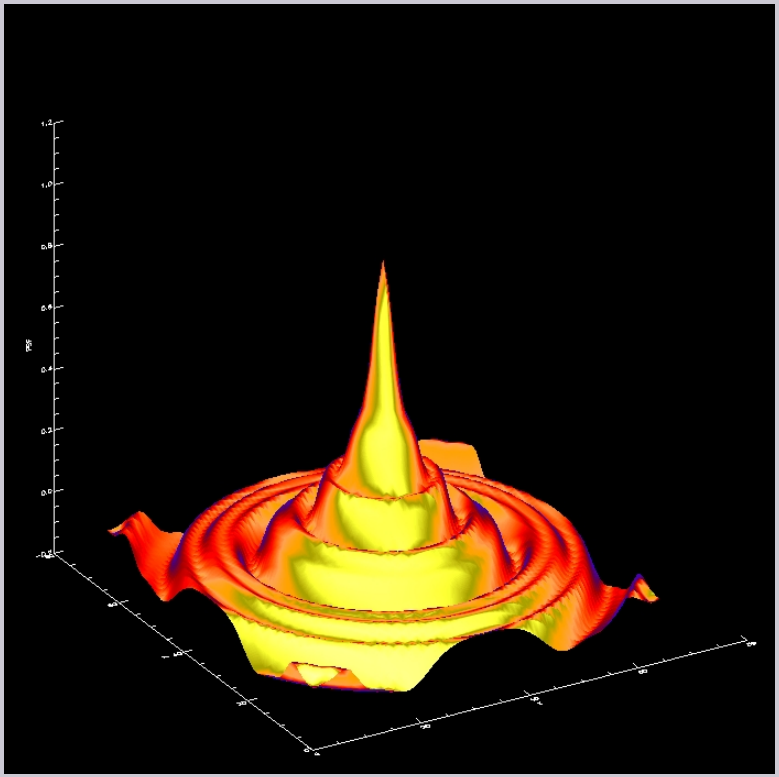


Approximation of background map

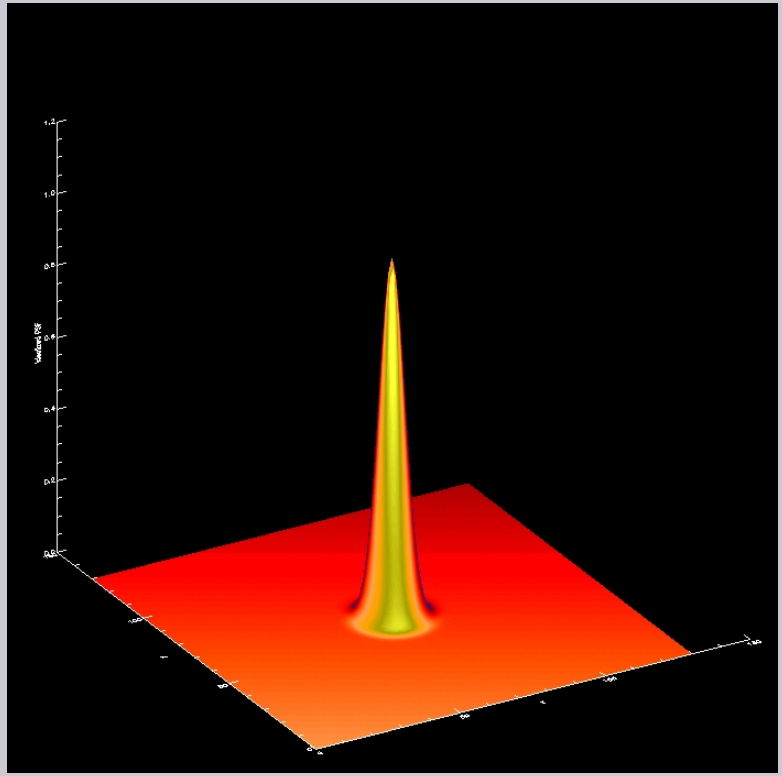
$$B(x,y) \simeq B'(x,y) := \frac{R(x,y)}{\sum_{n=1}^{N_{pix}} K(x_n, y_n)\delta x \delta y}$$

Solution map

RHESSI Experimental PSF



Idealized PSF $K^I(x, y)$

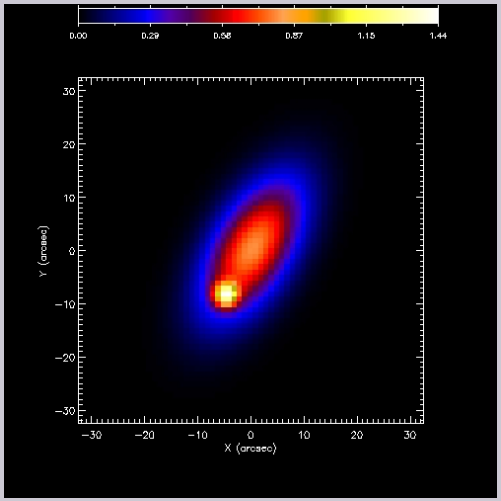


CLEAN map

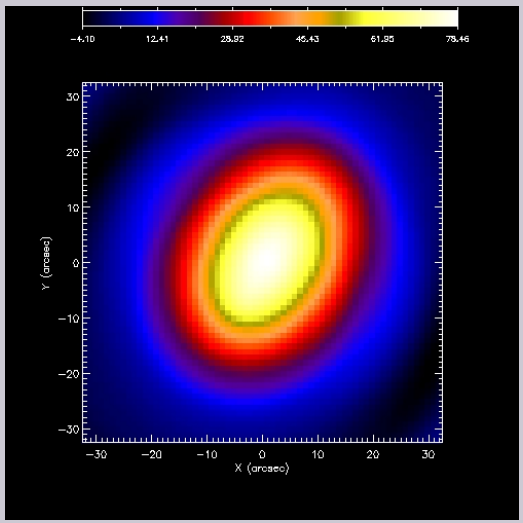
$$I^C(x, y) = \Big((CC + B) * K^I \Big) (x, y)$$

Results

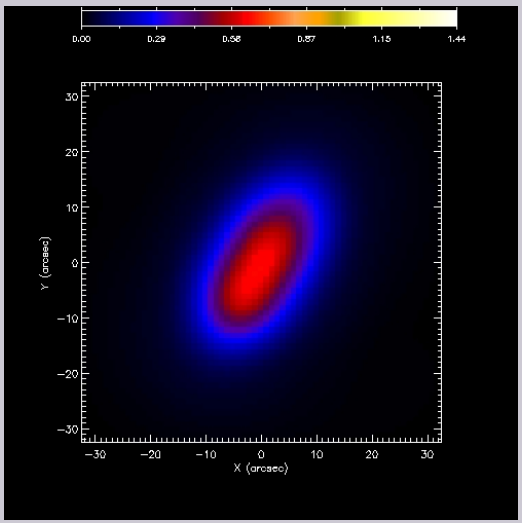
Simulated map



Dirty map



CLEAN solution map



Multiscale CLEAN

Requirements

$$K(x,y) = \sum_{j=1}^{N} K_j(x,y) \tag{6}$$

$$I^D(x,y) = \sum_{j=1}^{N} I_j^D(x,y) \tag{7}$$

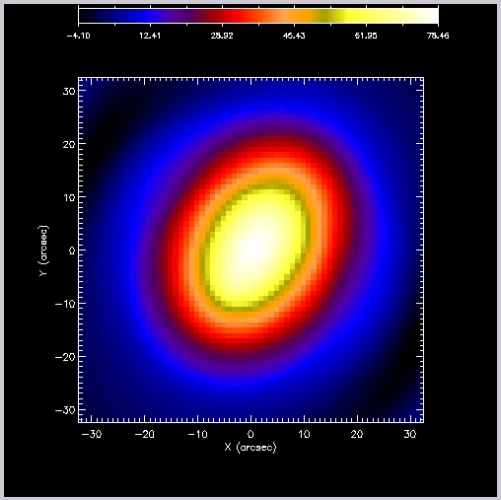
Solution model

$$I(x,y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i} m_i(x - x_{q_i}, y - y_{q_i}) + B(x,y) \tag{8}$$

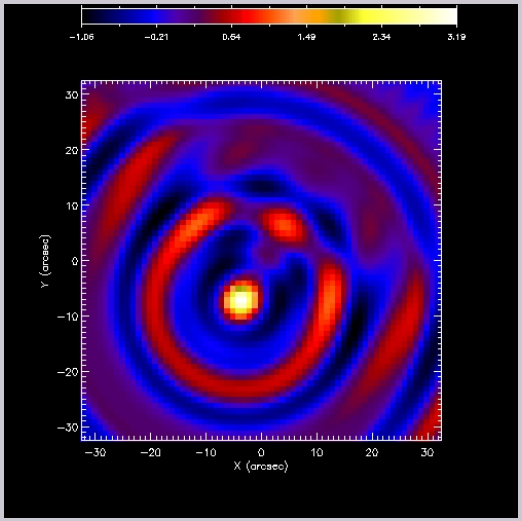
$$I_j^D(x,y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i} \left(m_i * K_j\right)(x - x_{q_i}, y - y_{q_i}) + \left(B * K_j\right)(x,y) \quad j = 1, \ldots, N \tag{9}$$

Multiscale CLEAN

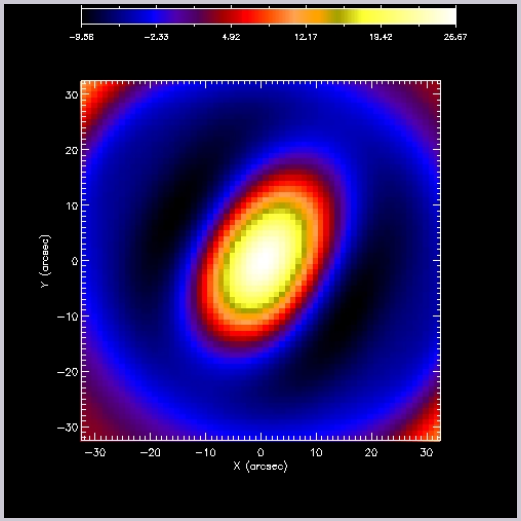
$I^D(x, y)$



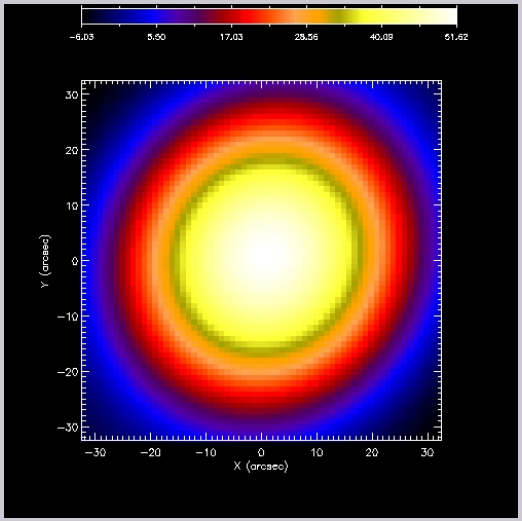
$I_1^D(x, y)$



$I_2^D(x, y)$



$I_3^D(x, y)$



Multiscale CLEAN loop (t-th iteration)

1. $I_1^D(x,y), \ldots, I_N^D(x,y)$

2. $M_j = \max(I_j^D(x,y)) \quad j = 1, \ldots, N$

3. $M_l = \max(M_1, \ldots, M_N) \qquad I_l^D(x_t, y_t) = M_l$

   in the position (x_t, y_t) there is a source with scale l

4. $CC(x,y) \leftarrow CC(x,y) + F_t \cdot m_l(x - x_t, y - y_t)$

5. $I_j^D(x,y) \leftarrow I_j^D(x,y) - F_t \cdot (m_l * K_j)(x - x_t, y - y_t) \quad \forall j = 1, \ldots, N$

STOP when there are no more sources

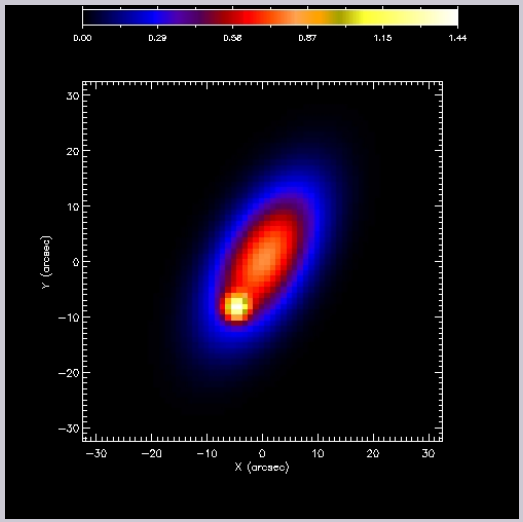
Multiscale CLEAN loop output

Residual maps

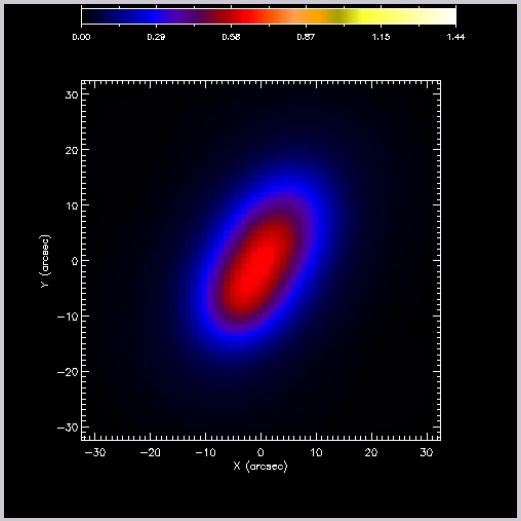
$$R_j(x,y) = (B * K_j)(x,y) \quad j = 1, \ldots, N$$

$$R(x,y) = \sum_{j=1}^{N} R_j(x,y) = (B * K)(x,y) \Rightarrow B(x,y) \simeq \frac{R(x,y)}{\sum_{n=1}^{N_{pix}} K(x_n, y_n)\delta x \delta y}$$

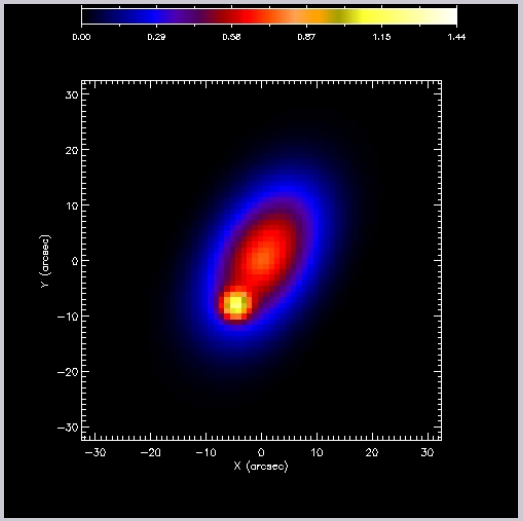
Original map



CLEAN

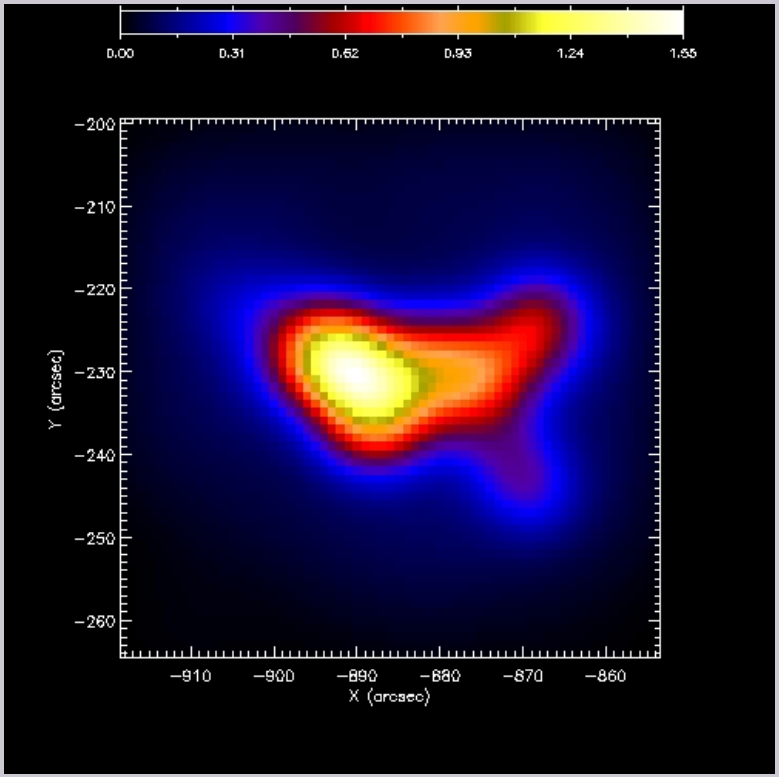


Multiscale CLEAN

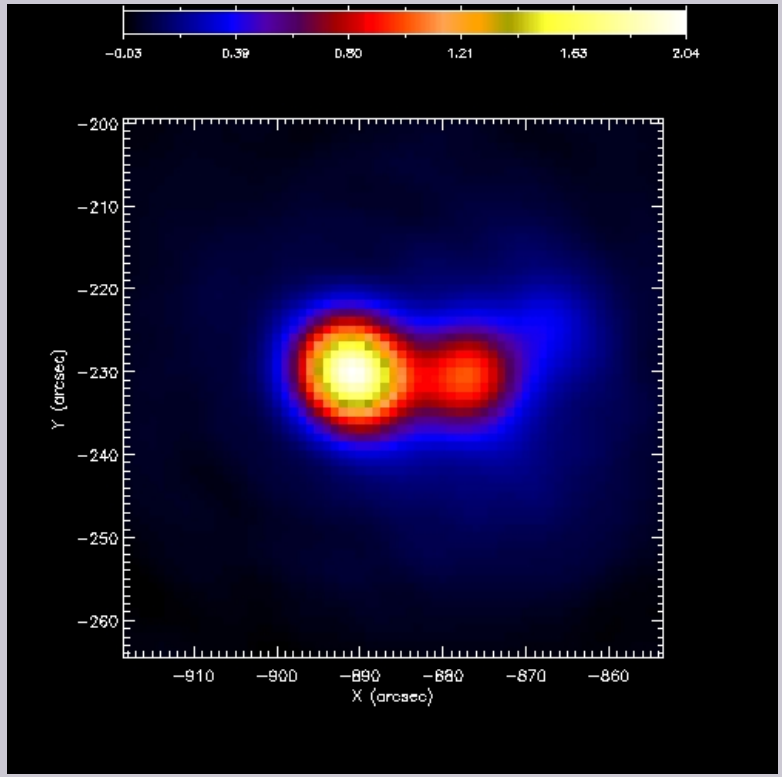


Results: July 23, 2002 - Energy range 28 - 32 keV

CLEAN

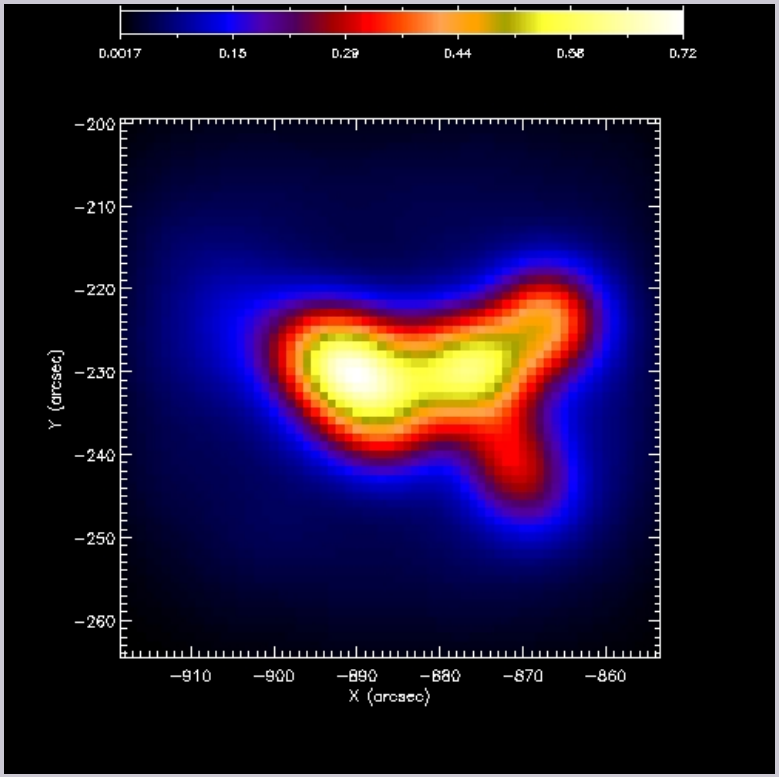


CLEAN multiscale

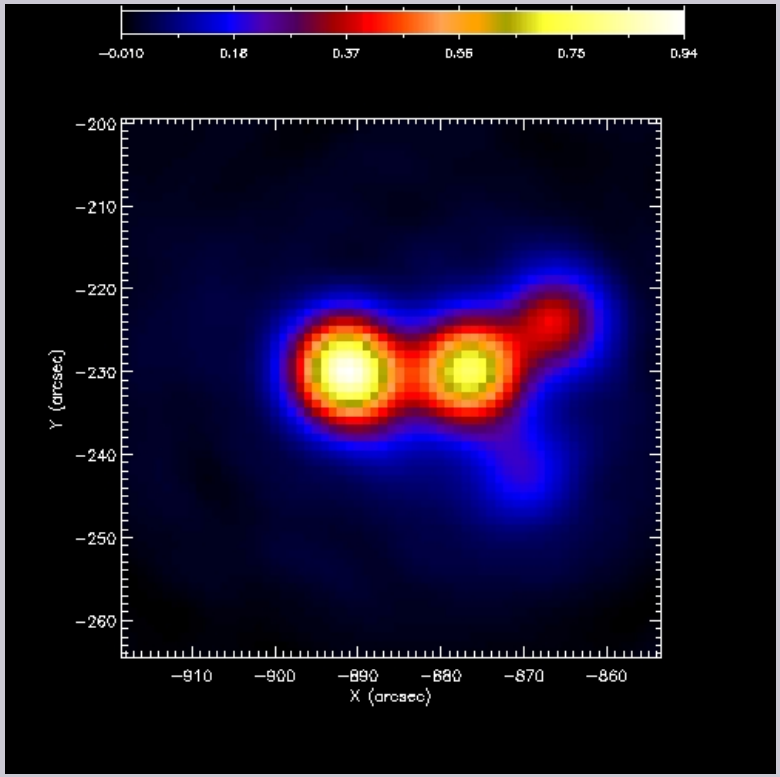


Results: July 23, 2002 - Energy range 32 - 36 keV

CLEAN

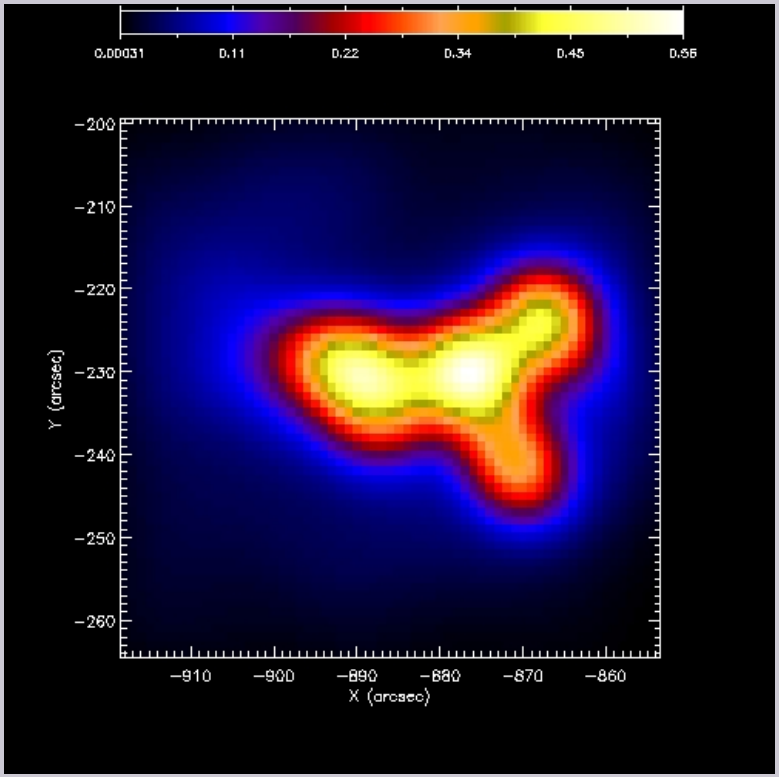
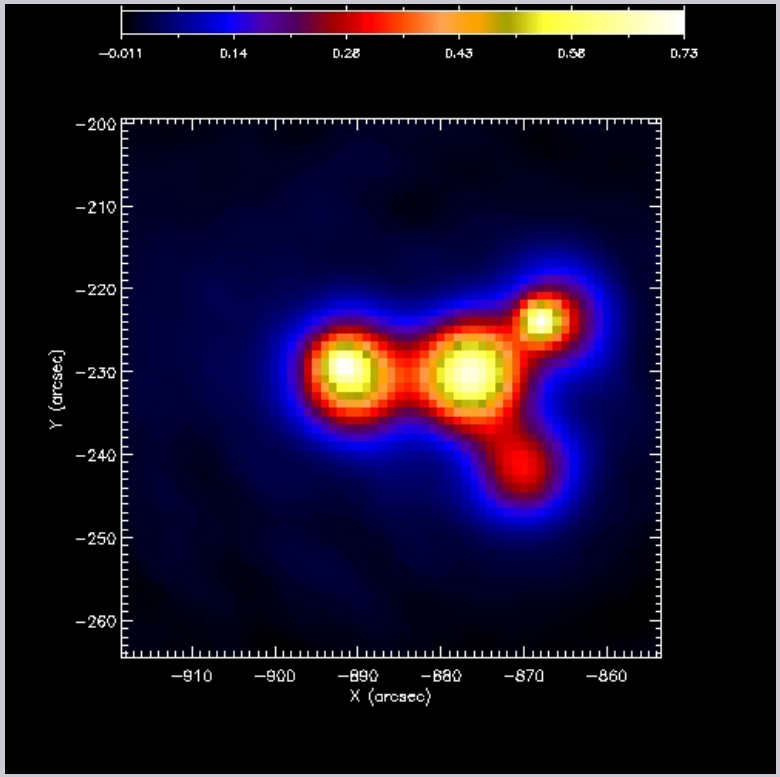


CLEAN multiscale



Results: July 23, 2002 - Energy range 36 - 41 keV

CLEAN



CLEAN multiscale