A (hopefully) friendly introduction to Isogeny-Based Cryptography

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June 29, 2023

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Purpose and Terminology

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In order to carry out encryption and decryption, we need so-called cryptographic keys.

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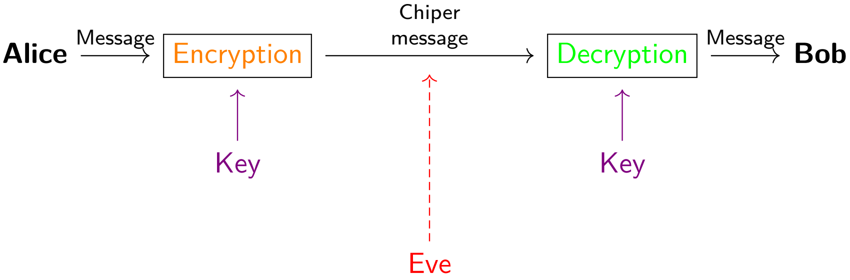

Types and keys

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• Key Exchange Problem: how can two parties exchange keys in such a way as to establish a secure communication channel?

Diffie-Hellman Key Exchange

Diffie-Hellman Key Exchange (DHKE), 1976

1. Alice and Bob publicly agree on a cyclic finite group G and a generator g.
2. Alice chooses $a \in \{1, \ldots, \mathrm{ord}(G)\}$, computes g^a and sends it to Bob. Her secret key is a.
3. Bob chooses $b \in \{1, \ldots, \mathrm{ord}(G)\}$, computes g^b and sends it to Alice. His secret key is b.
4. Alice computes $(g^b)^a = g^{ba}$.
5. Bob computes $(g^a)^b = g^{ab}$.

The secret common key is $g^{ba} = g^{ab}$.

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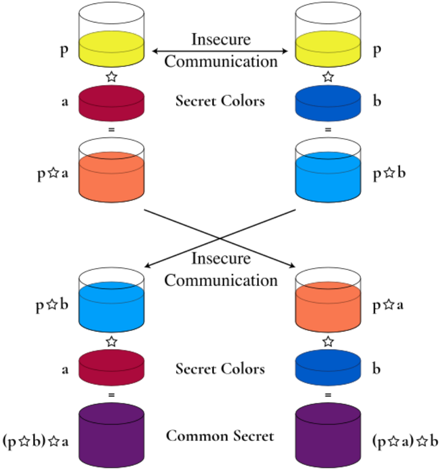
- Diffie-Hellman Problem (DHP): Let G be a finite cyclic group and let g be a generator. Given g^a and g^b, find g^{ab}.

Diffie-Hellman Key Exchange



[Picture from Borradaile, G. "Defend Dissent." Corvallis: Oregon State University, 2021.]

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- Others

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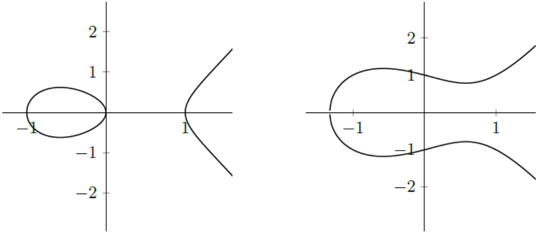
Elliptic Curves

An *elliptic curve* is a pair (E, O_E), where E is a nonsingular projective curve of genus 1 and $O_E \in E$ is a fixed point.



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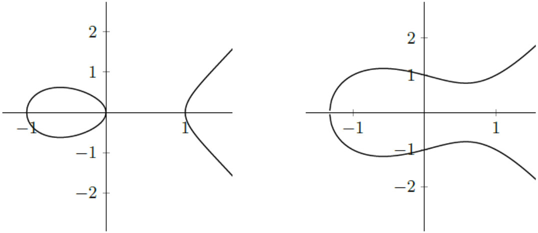

(a) $y^2 = x^3 - x$ (b) $y^2 = x^3 - x + 1$

Weierstrass form: $y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$

If $\mathrm{char}(k) \neq 2, 3$: $y^2 = x^3 + Ax + B$

The *discriminant* of E is $\Delta(E) = -(4A^3 + 27B^2)$.

Elliptic curves

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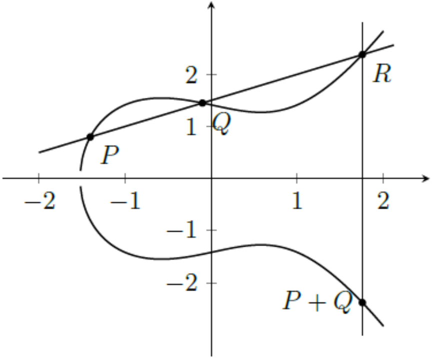
Properties

- A curve given by a Weierstrass equation is nonsingular if and only if $\Delta(E) \neq 0$.
- Two elliptic curves are isomorphic over $\overline{k}$ if and only if they have the same j-invariant.
- Let $j_0 \in \overline{k}$. There exists an elliptic curve defined over $k(j_0)$ whose j-invariant is j_0.

Group law:



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An example of isogeny is the *multiplication-by-m* with $m \in \mathbb{Z}$:

$$\begin{aligned}[m]\colon E &\to E \\ P &\mapsto P + \cdots + P\end{aligned}$$

Definition

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Theorem

Let $\varphi\colon E \to E'$ be an isogeny of degree ℓ. Then there exists an isogeny $\widehat{\varphi}\colon E' \to E$ of degree ℓ, called *dual isogeny*, such that

$$\varphi \circ \widehat{\varphi} = [\ell] \quad \text{and} \quad \widehat{\varphi} \circ \varphi = [\ell].$$

Ordinary and Supersingular EC

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- **Tate's Theorem:** If two elliptic curves are isogenous, then they are of the same type.
- The endomorphism ring of an ordinary elliptic curve is commutative. The endomorphism ring of a supersingular elliptic curve is noncommutative.

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Underlying Problems

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- General Isogeny Problem: Given two isogenous elliptic curves, find an isogeny between them.
- ℓ-Isogeny Problem: Given two ℓ-isogenous elliptic curves, find an ℓ-isogeny between them.

Isogeny graphs

Definition

Let ℓ be a prime number such that $\ell \neq \mathrm{char}(k)$.
An *ℓ-isogeny graph $G_\ell(k)$* is a graph whose vertices are j-invariants of elliptic curves defined over k and whose edges are ℓ-isogenies defined over k between them.

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It follows from Tate's theorem that the graph $G_\ell(k)$ can always be partitioned into ordinary and supersingular components.

Ordinary case

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Definition

An *ℓ-volcano* is a connected undirected graph whose vertices are partitioned into one or more *levels* $V_0, \ldots, V_d$ such that:

(i) the subgraph on V_0 (the *surface*) is a regular graph of degree at most 2;

(ii) for $i > 0$, each vertex in V_i has exactly one neighbor in level V_{i-1};

(iii) for $i < d$, each vertex in V_i has degree $\ell + 1$.

We call d the *depth* of the volcano and we call V_d the *floor*.

Ordinary Case

- V_0 regular graph of degree at most 2;
- each vertex in V_i has exactly one neighbor in V_{i-1}, for $i > 0$;
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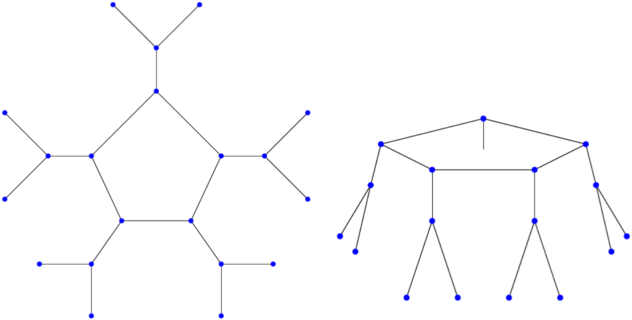

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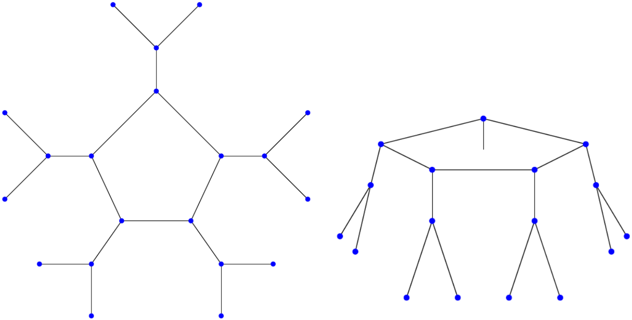
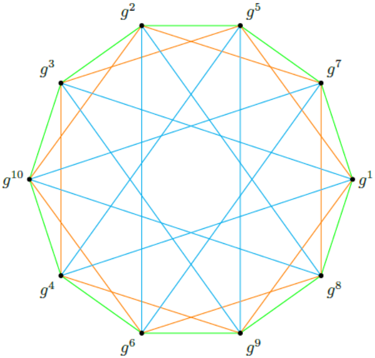


- An ordinary component of $G_\ell(\mathbb{F}_q)$ is an ℓ-volcano.

An example



- V = {set of generators of a cyclic group of order 11};
- $S = \{3, 5, 7, 3^{-1}, 5^{-1}, 7^{-1}\} \subseteq (\mathbb{Z}/11\mathbb{Z})^\times$.

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  4. Alice computes $g_{AB} = \rho_A(g_B)$ and Bob computes $g_{AB} = \rho_B(g_A)$.

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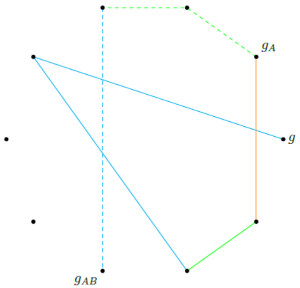

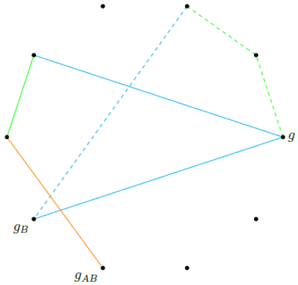

The order of the steps in a route does not matter: what counts is only how many times each element of D appears in the route.

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N.B. The cryptosystem works because we are in a commutative environment.

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2022: Castryck and Decru use these extra information to broke the cryptosystem.

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Restriction to $\mathbb{F}_p$

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N.B. The problem with the RS protocol on ordinary elliptic curves is that it takes several minutes per key exchange. In the supersingular case this efficiency problem does not occur!

Thanks for your attention!