The problem of a combinatorial classification of finite metric spaces via their fundamental polytopes was suggested by Vershik in 2010, but to date the structure of these polytopes (even their face numbers) remains largely unknown. In this talk, based on joint work with Linard Hoessly, I will explain how to associate a hyperplane arrangement to every split pseudometric and how to use its combinatorics in order to compute the face numbers of fundamental polytopes and Lipschitz polytopes of tree-like metrics. I will discuss how to apply our results to specific examples and, time permitting, I will briefly comment on the potential of our model beyond tree-like metrics. I will introduce all the mentioned mathematical objects so that no specific prerequisites should be required - hence all graduate students are welcome!