Spectral theory of vector and tensor fields on Schwarzschild spacetime

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Abstract After a separation of variables ansatz, the equations for vector and tensor test fields on a Schwarzschild black hole background reduce to radial ODEs. More precisely, we refer to the wave equation for vector fields and the Lichnerowicz equation for metric perturbations. These equations can be thought of as the harmonic gauge versions of, respectively, the Maxwell and linearized Einstein equations. These radial ODEs pose a spectral problem that is non standard in several ways they are coupled matrix equations, they are not naturally self adjoint on a function space with a positive definite inner product, and the spectral parameter appears non linearly both linearly and quadratically. Nevertheless, we can realize these differential equations as closed operators between certain Hilbert spaces, with purely real spectrum in the sense of operator pencils. This result is only possible due to an explicit and highly non trivial equivalence of these equations to systems of decoupled generalized Regge Wheeler scalar equations, which are known to pose standard self adjoint spectral problems.