IMAGING OF UNKNOWN TARGETS INSIDE INHOMOGENEOUS BACKGROUNDS BY MEANS OF QUALITATIVE INVERSE SCATTERING

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ABSTRACT. In this paper a new formulation of the Linear Sampling Method, called the no–Sampling Linear Sampling Method, is applied to the imaging and detection of unknown scatterers located inside an inhomogeneous background. Namely, by following a previous work by Colton and Monk, a modified far–field equation is used, which allows one for using line current sources and near–field measurements. The Green’s function of the inhomogeneous background is numerically computed and used as the right hand side of the modified far–field equation. The proposed method is then applied to two different scenarios: the detection of breast tumors and the imaging of cracks inside a dielectric slab. A numerical analysis of the method capabilities is performed when the model parameters are not exactly known.

1. Introduction

In the last years there has been a growing interest in inverse scattering problems arising in microwave diagnostics, which finds application in several fields, ranging from non-destructive testing and evaluation to medical imaging, and from civil engineering to target identification [9, 13, 1, 11, 19, 4]. Basically, methods for the solution of inverse scattering problems can be grouped into two families. Quantitative methods aim at determining the point values of the electrical parameters of the targets. This can be accomplished by means of non-linear optimization schemes [12] which obtain stable approximations of the refractive index by stopping an iterative procedure appropriately initialized; or by solving the inverse scattering problem under the Born or Rytov approximation [10] by using restoration techniques formulated within the framework of the regularization theory for linear inverse problems. Common advantageous features of these

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techniques are the rich informative content of the restored map and a notable accuracy of the reconstruction. However, these methods have two relevant drawbacks: they converge only if carefully initialized and are typically computationally expensive.

On the other hand, qualitative methods provide a criterion to decide whether or not a point is in the support of the inhomogeneity. Historically, the first example of visualization method is the Linear Sampling Method \cite{5}, which provides images of the profile of a scatterer by plotting all the points where the regularized solution of a linear Fredholm equation of the first kind blows up. After that, other visualization methods have been formulated, most requiring a factorization scheme for integral operators related to the far-field operator \cite{15, 16, 17}. All of these methods share the advantages that no a priori information on the scatterer is required for their application and that they present a highly computational efficiency.

In this paper we describe the application of a no-sampling formulation of linear sampling \cite{2} for detecting inhomogeneities inside structures.

The no-sampling implementation is based on a functional formulation of the traditional Linear Sampling Method \cite{10} on a direct sum of Hilbert spaces which allows a unique regularization procedure. This means that, opposite of the classic approach, only one regularization parameter is needed and this increases notably the computational speed of the inversion algorithm without deterioration of the reconstruction.

The contributions of the present paper are essentially two. First, the no-sampling Linear Sampling Method (nLSM) is applied for detecting unknown scatterers located inside an exactly known inhomogeneous background. Namely, by following \cite{5} and a previous work by Colton and Monk \cite{3}, a modified far-field equation is used, which allows one for using line current sources and near-field measurements. From a mathematical viewpoint, the presence of the inhomogeneous background is taken into account by its Green’s function, which is numerically computed by the Method of Moments \cite{14} and used as the right hand side of the modified far-field equation.

Then, we study the performances of this approach when the model parameters are not exactly known and therefore only an approximate version of the background Green’s function is utilized in the code. In this study, we consider two different scenarios, i.e. the detection of breast tumors in a microwave tomography setting and the imaging of cracks inside a dielectric slab, and we perform a numerical analysis in order to test the robustness of the method when the background parameters are both underestimated and overestimated.

The paper is organized as follows: In Section 2 the mathematical formulation of the problem is recalled and in Section 3 some numerical tests are presented. Finally, some conclusions are drawn in Section 4.

2. Mathematical set-up

Let us consider a finite set of non-magnetic cylindrical scatterers, whose axes are parallel to the z-axis of the reference frame and immersed in the vacuum, which is characterized by the dielectric permittivity $\varepsilon_0$ and the magnetic permeability $\mu_0$. Moreover, the cross sections of the scatterers are assumed to be bounded in the plane.

As it is well known \cite{14, 7}, if the incident field is TM polarized with respect to the axes of the targets under test and $E_{inc}$ denotes the z-component of the incident electric field, then the z-component $E$ of the total electric field satisfies the Lippmann–Schwinger integral equation

$$
E(r) = E_{inc}(r) - i\omega\mu_0 \int_{\mathbb{R}^2} \tau(r') E(r' | r') G_0(r | r') dr', \quad \forall r \in \mathbb{R}^2.
$$

In equation (1) $\omega$ is the operating angular frequency and $\tau(r) = \omega(\varepsilon_0 - \varepsilon(r))$, for any $r \in \mathbb{R}^2$, denotes the scattering potential with respect to the vacuum, being $\varepsilon(r)$ the dielectric permittivity at point $r$.

Since $\tau$ has compact support, the integration domain in equation (1) can be restricted to any region containing the support of the scatterers. Finally, $G_0(r | r') = \frac{1}{4} H^{(1)}_0(k_0 | r - r'|)$, for any $r, r' \in \mathbb{R}^2$, is the Green’s function for free-space configuration \cite{33}, being $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ the wavenumber in the vacuum.

Since we are interested in determining anomalies inside known structures, it is useful to generalize the integral equation (1) to the case of targets located in an inhomogeneous background.

Let then $\varepsilon_B(r), r \in \mathbb{R}^2$, be the relative dielectric permittivity of the inhomogeneous background at the position $r$ and, for the sake of simplicity, assume that there exists a circle $C$ of radius $R_B$ such that $\varepsilon_B(r) = \varepsilon_0$, for $r = |r| > R_B$. The related Green’s function $G_B(\cdot | r')$ solves then the
Helmholtz equation
(2) \[ \nabla^2 G(r | r') + k^2(r) G(r | r') = \delta(r - r') \]
for each \( r' \in \mathbb{R}^2 \), being \( k(r) = \omega \sqrt{\varepsilon_B(r)} \mu_0 \), and satisfies the radiation condition
(3) \[ \lim_{r \to \infty} \sqrt{r} \left( \frac{\partial G_B}{\partial r} - i k_0 G_B \right) = 0. \]

If \( T \subset \mathbb{R}^2 \) denotes a region containing the support \( D \) of all the inspected scatterers, the relation of interest can then be written as follows
(4) \[ E(r) = E_{B,\text{inc}}(r) - \omega \mu_0 \int_T \tau_B(r') E(r') G_B(r | r') dr', \ \forall r \in \mathbb{R}^2, \]
being \( \tau_B(r) = i \omega (\varepsilon_B(r) - \varepsilon_0) \) the scattering potential with respect to the reference background and \( E_{B,\text{inc}} \) the electric field measured when the scatterers are not present. Accordingly, \( E_{B,\text{inc}} \) satisfies the integral equation
(5) \[ E_{B,\text{inc}}(r) = E_{\text{inc}}(r) - \omega \mu_0 \int_C \tau(r') E_{B,\text{inc}}(r') G_0(r | r') dr', \ \forall r \in \mathbb{R}^2. \]

The inverse problem we are interested in is to determine the support of inhomogeneities inside the region under investigation from measurements of the scattered field \( E_{B,\text{scatt}} := E - E_{B,\text{inc}} \) on a circle \( \Gamma_M \) of prefixed radius \( a > 0 \), i.e., \( \Gamma_M := \{ r \in \mathbb{R}^2 : |r| = a \} \). The idea pursued in this paper follows [5][8] and consists of applying the nLSM to a generalized version of equation (4) which has the same structure as the classical equation (7).

In the following, we will suppose that the sources are line currents parallelly oriented along the \( z \)-axis and located on a circle \( \Gamma_S \) of radius \( b > 0 \), i.e., \( \Gamma_S := \{ r \in \mathbb{R}^2 : |r| = b \} \). As a consequence, the incident field \( E_{\text{inc}} \), at the position \( r \in \mathbb{R}^2 \), can be written as
(6) \[ E_{\text{inc}}(r, d) = -\frac{\omega \mu_0}{4} I H_0^{(1)}(k_0 |r - d|), \]
being \( d \in \Gamma_S \) the position of the source and \( I \) the complex amplitude of its current.

It is worth remarking that the dependence of the incident field \( E_{\text{inc}} \) on the position of the source has been indicated. Analogously, in the following, the dependence on the position of the transmitting antenna of all the involved electric fields will be always made explicit.

The first step for the description of the imaging method is the introduction of the modified far field operator \( F : L^2(\Gamma_S) \to L^2(\Gamma_M) \) defined as
(7) \[ [Fg(\cdot)](r) := \int_{\Gamma_S} E_{B,\text{scatt}}(r, d) g(d) ds(d), \ \forall r \in \Gamma_M. \]

Now, for each \( z \in \mathbb{R}^2 \), we introduce the modified far-field equation
(8) \[ [Fg_k(\cdot)](r) = G_B(r | z), \ \forall r \in \Gamma_M, \]
where \( g_k(\cdot) \in L^2(\Gamma_S) \). Note that equation (5) is an ill–posed Fredholm integral equation of the first kind and hence a solution \( g_k(\cdot) \) may not exist. However, a theorem in [8] assures us that there exists an approximate solution \( g_k(\cdot) \) whose \( L^2 \)–norm blows up to infinity when \( z \) approaches the boundary of the inhomogeneity with respect to the reference background.

In practical applications the scattered data are affected by measurements noise, and therefore, from a mathematical point of view, we have at disposal only a noisy version \( E_{B,\text{scatt}}^H := E_{B,\text{scatt}} + H \) of the integral kernel, where \( H \) denotes the noise. An estimate of the noise level can be provided by the non–negative real number \( h \) such that \( \|F^h - F\| \leq h \), being \( \| \cdot \| \) the standard operator norm, and \( F^h \) the noisy version of \( F \).

In real experiments, the incident field is emitted from a finite set of antennas and the scattered field is measured by a finite set of receivers. As a consequence, \( \Gamma_M \) and \( \Gamma_S \) must be replaced by discrete sets of points. For the sake of simplicity, hereafter we will assume that \( N \) receivers are uniformly spaced on \( \Gamma_M \), and \( N \) transmitters are uniformly spaced on \( \Gamma_S \). Accordingly, the positions of the measurements points and the sources can be written as follows, respectively, \( r_i = \left( a \cos \frac{2\pi(i-1)}{N}, a \sin \frac{2\pi(i-1)}{N} \right) \), for \( i = 1, \ldots, N \), and \( d_j = \left( b \cos \frac{2\pi(j-1)}{N}, b \sin \frac{2\pi(j-1)}{N} \right) \), for \( j = 1, \ldots, N \). This leads to the introduction of a \( N \times N \) complex matrix \( F^H \) with entries
(9) \[ (F^H)_{ij} := \frac{2\pi}{N} E_{B,\text{scatt}}^H(r_i, d_j), \ \forall i, j = 1, \ldots, N, \]
which represents the discretization of the operator \( F^h \).
The implementation of the no-sampling linear sampling suggested in \cite{2} can be summarized as follows. Consider the Hilbert space $[L^2(T)]^N$ equipped with the scalar product
\begin{equation}
(F(\cdot), G(\cdot))_{2,N} := \sum_{i=1}^{N} \int_{T} (f_i(z), g_i(z))_{L^2} \, dz
\end{equation}
for all $f(\cdot) := \{f_i(\cdot)\}^N_{i=1}$ and $g(\cdot) := \{g_i(\cdot)\}^N_{i=1}$ elements of $[L^2(T)]^N$, as well as the induced norm $\| \cdot \|_{2,N}$. Here $(\cdot, \cdot)_N$ is the scalar product induced by the above discretization, and it is defined as
\begin{equation}
(\cdot, \cdot)_N := \langle \cdot, \cdot \rangle := \frac{2\pi}{N} \sum_{i=1}^{N} w_i^T \overline{w}_i \in [0, +\infty),
\end{equation}
where $w_i^T$ denotes the transpose of $w_i$ and $\overline{w}_i$ the complex conjugate of $w_i$.

Define the linear operator $F^h : [L^2(T)]^N \to [L^2(T)]^N$ given by
\begin{equation}
[F^h g(\cdot)](\cdot) := \left\{ \sum_{j=1}^{N} (P^H)_{ij} g_j(\cdot) \right\}^N_{i=1}
\end{equation}
\begin{equation}
\forall g(\cdot) := \{g_j(\cdot)\}^N_{j=1} \in [L^2(T)]^N.
\end{equation}

In the no-sampling approach the role of equation \cite{3} is played by the following equation in $[L^2(T)]^N$
\begin{equation}
[F^h g(\cdot)](\cdot) = G_B(\cdot),
\end{equation}
where $G_B(\cdot) := (G_B(r_1 \mid \ldots, G_B(r_{\gamma} \mid \ldots))$.

Although the problem of determining the generalized solution of the functional equation \cite{19} is well-posed, we know that such an inverse problem is, in general, ill-conditioned and therefore a regularization method is anyway necessary. If $\sigma_H > 0$ is the rank of the data matrix $P^H$ and $\{\sigma_H^p, \gamma_H^p\}^N_{p=1}$ its singular system, it can be shown, similarly as in \cite{2}, that the Tikhonov regularized solution of equation \cite{19} is given by
\begin{equation}
g_\alpha(\cdot) = \frac{N}{2\pi} \sum_{p=1}^{\sigma_H} \frac{\sigma_H^p}{(\sigma_H^p)^2 + \alpha} (G_B(\cdot), \gamma_H^p)_N u_H^p,
\end{equation}
where $(G_B(\cdot), \gamma_H^p)_N$ is the element in $L^2(T)$ defined for all $z \in T$ as $(G_B(z), \gamma_H^p)_N$, while $\alpha$ is a real positive number, which plays the role of regularization parameter. $\alpha$ can be determined, according to the generalized discrepancy principle, as the zero of the generalized discrepancy function \cite{2}
\begin{equation}
\rho(\alpha) := \| [F^h g_\alpha(\cdot)](\cdot) - G_B(\cdot) \|_{2,N}^2 - h^2 \| g_\alpha(\cdot) \|_{2,N}^2.
\end{equation}

If $\alpha^*$ is the optimal value provided by this criterion, i.e. $\rho(\alpha^*) = 0$, then $g_\alpha^*(\cdot)$ is the optimal regularized solution of problem \cite{13}.

It is worth remarking that, thanks to the singular system properties, it results
\begin{equation}
\| g_\alpha(\cdot) \|^2_{2,N} = \frac{N^2}{4\pi} \sum_{p=1}^{\sigma_H} \frac{\sigma_H^p}{(\sigma_H^p)^2 + \alpha}^2 |(G_B(\cdot), \gamma_H^p)_N|^2.
\end{equation}
\begin{equation}
\| g_\alpha(\cdot) \|^2_{2,N} = -\log \left( (\| g_\alpha(\cdot) \|^2_{2,N})^2 \right).
\end{equation}

3. Numerical results

The visualization algorithm described in the previous section has been assessed against synthetic near-fields noisy data. In all the considered cases, the scattered data and the Green’s function of the inhomogeneous background are computed by means of a Method of Moment code \cite{13}. Furthermore, the scattered data are corrupted by additive Gaussian noise with zero mean and power such that the resulting signal–to–noise ratio (SNR) is equal to 30 dB. In all the numerical tests the operating frequency is equal to 1GHz and the values of indicator function \cite{17} have been reported in the range $[0, 1]$ by an affine transformation.

The first simulation consists in detecting a tumor inside a simple breast phantom shown in Figure \cite{1}. The breast model is composed by a homogeneous circular disk with radius $R^\text{rad} = 0.04$ m representing the fat surrounded by a concentric ring of width $w^{\text{ring}} = 0.002$ m modeling the
skin. The dielectric parameters of the fat are \( \varepsilon_{\text{fat}} = 40.9 \) and \( \sigma_{\text{fat}} = 0.9 \) S m\(^{-1} \), whereas the skin is characterized by \( \varepsilon_{\text{skin}} = 10 \) and \( \sigma_{\text{skin}} = 0.15 \) S m\(^{-1} \). Moreover, the model includes two circular glands located inside the fat with relative dielectric permittivity \( \varepsilon_{\text{gland}} = 11.5 \) and electric conductivity \( \sigma_{\text{gland}} = 0.17 \) S m\(^{-1} \) and radii \( R_{1} = 0.01 \text{ m} \) and \( R_{2} = 0.015 \text{ m} \). The inspected tumor is modeled as a circular disk of radius \( R_{\text{tumor}} = 0.003 \text{ m} \), relative dielectric permittivity \( \varepsilon_{\text{tumor}} = 53.9 \) and electric conductivity \( \sigma_{\text{tumor}} = 0.7 \) S m\(^{-1} \), centered at the point of Cartesian coordinates \( (x_{\text{tumor}}, y_{\text{tumor}}) \), \( x_{\text{tumor}} = 0.02 \) m, \( y_{\text{tumor}} = 0 \) m.

Two groups of numerical tests have been performed to validate the robustness of the proposed algorithm when the model is not exactly known, i.e., when the electromagnetic parameters of the fat and the skin in the model are not the right ones.

In the former group, the tumor is inspected by using the exact values of the relative permittivity and conductivity of thefat, whereas the parameters of \( \varepsilon_{\text{skin}} \) and \( \sigma_{\text{skin}} \) are assumed to be different from those used to solve the forward problem. Namely, all the cases corresponding to the couples of relative dielectric permittivities and electric conductivity \( (\varepsilon_{\text{fat}}, \sigma_{\text{fat}}) \in \{36.81, 40.9, 44.9\} \times \{0.81 \text{ S m}^{-1}, 0.9 \text{ S m}^{-1}, 0.99 \text{ S m}^{-1}\} \) have been simulated and the obtained results are reported in Figure 2. Note that 40.9 and 0.9 S m\(^{-1} \) are the exact values of the relative permittivity and conductivity of the skin, respectively.

In the latter group, presented in Figure 3, the Green’s function is computed by using the exact values of the relative permittivity and conductivity of the skin, and considering all cases corresponding to \( (\varepsilon_{\text{fat}}, \sigma_{\text{fat}}) \in \{9, 10, 11\} \times \{0.135 \text{ S m}^{-1}, 0.15 \text{ S m}^{-1}, 0.165 \text{ S m}^{-1}\} \). The results are presented in Figure 3. Note that 10 and 0.15 S m\(^{-1} \) are the exact values of the relative permittivity and conductivity of the fat, respectively.

Tables 1 and 2 present the localization error

\[
d := \sqrt{(x_{\Psi}^{\text{max}} - x_{\text{tumor}})^2 + (y_{\Psi}^{\text{max}} - y_{\text{tumor}})^2},
\]

being \( (x_{\Psi}^{\text{max}}, y_{\Psi}^{\text{max}}) \) the Cartesian coordinates of the point where the indicator function \( \Psi \) is maximum, for the two analysis.

The second test consists in retrieving an empty hole inside a dielectric slab. The intact structure of reference is composed by a non conductive square slab characterized by a relative dielectric permittivity \( \varepsilon_{\text{fat}} = 4 \). Such a structure is centered at the origin of the used reference system and its sides are \( L_{x} = L_{y} = 0.2 \) m long. The crack is supposed to be a rectangle of sides 0.01 m and 0.03 m and centered at the point of Cartesian coordinates \( (0, -0.025) \). The exact model is presented in Figure 4.
Figure 2. Indicator function obtained by using a breast model with the exact fat parameters ($\varepsilon_r^{fat} = 10$, $\sigma_r^{fat} = 0.15$ S m$^{-1}$) and with (a) $\varepsilon_r^{skin} = 36.81$, $\sigma_r^{skin} = 0.81$ S m$^{-1}$; (b) $\varepsilon_r^{skin} = 36.81$, $\sigma_r^{skin} = 0.9$ S m$^{-1}$; (c) $\varepsilon_r^{skin} = 36.81$, $\sigma_r^{skin} = 0.99$ S m$^{-1}$; (d) $\varepsilon_r^{skin} = 40.9$, $\sigma_r^{skin} = 0.81$ S m$^{-1}$; (e) $\varepsilon_r^{skin} = 40.9$, $\sigma_r^{skin} = 0.9$ S m$^{-1}$; (f) $\varepsilon_r^{skin} = 40.9$, $\sigma_r^{skin} = 0.99$ S m$^{-1}$; (g) $\varepsilon_r^{skin} = 44.99$, $\sigma_r^{skin} = 0.81$ S m$^{-1}$; (h) $\varepsilon_r^{skin} = 44.99$, $\sigma_r^{skin} = 0.9$ S m$^{-1}$; (i) $\varepsilon_r^{skin} = 44.99$, $\sigma_r^{skin} = 0.99$ S m$^{-1}$.

Table 1. Localization error $d$ (exact parameters of the skin)

<table>
<thead>
<tr>
<th>$\varepsilon_r^{fat}$</th>
<th>$\varepsilon_r^{fat} = 9$</th>
<th>$\varepsilon_r^{fat} = 10$</th>
<th>$\varepsilon_r^{fat} = 11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r^{fat} = 0.135$ S m$^{-1}$</td>
<td>1.58 · 10$^{-3}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>2.83 · 10$^{-4}$</td>
</tr>
<tr>
<td>$\sigma_r^{fat} = 0.15$ S m$^{-1}$</td>
<td>1.58 · 10$^{-3}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>3.12 · 10$^{-4}$</td>
</tr>
<tr>
<td>$\sigma_r^{fat} = 0.165$ S m$^{-1}$</td>
<td>1.58 · 10$^{-3}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>3.33 · 10$^{-4}$</td>
</tr>
</tbody>
</table>

Table 2. Localization error $d$ (exact parameters of the fat)

<table>
<thead>
<tr>
<th>$\varepsilon_r^{fat}$</th>
<th>$\varepsilon_r^{fat} = 36.81$</th>
<th>$\varepsilon_r^{fat} = 40.9$</th>
<th>$\varepsilon_r^{fat} = 44.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r^{fat} = 0.81$ S m$^{-1}$</td>
<td>1.58 · 10$^{-3}$</td>
<td>1.58 · 10$^{-3}$</td>
<td>6.67 · 10$^{-5}$</td>
</tr>
<tr>
<td>$\sigma_r^{fat} = 0.9$ S m$^{-1}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>4.53 · 10$^{-5}$</td>
</tr>
<tr>
<td>$\sigma_r^{fat} = 0.99$ S m$^{-1}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>0.707 · 10$^{-3}$</td>
<td>2.53 · 10$^{-5}$</td>
</tr>
</tbody>
</table>

The investigation region $T$ is a square region centered at the origin whose side is 0.3 m long and the indicator has been computed in 100 · 100 points uniformly distributed inside $T$, $N = 36$ emitters and receivers uniformly spaced along two circles of radius 0.2 m and 0.15 m, respectively.
The robustness of the algorithm is assessed by using the actual electromagnetic parameters of the dielectric slab, whereas the sizes of the slab are affected by errors. Namely, the models corresponding to \((L_x, L_y) \in \{0.19, 0.20, 0.21\} \times \{0.19, 0.20, 0.21\}\) are used to compute the Green’s function appearing in the right hand side of equation (8).
Figure 5. Indicator function obtained by using a lab model model with the exact permittivity value and with (a) $L_x = 0.19 \, \text{m}$, $L_y = 0.19 \, \text{m}$; (b) $L_x = 0.19 \, \text{m}$, $L_y = 0.20 \, \text{m}$; (c) $L_x = 0.19 \, \text{m}$, $L_y = 0.19 \, \text{m}$; (d) $L_x = 0.20 \, \text{m}$, $L_y = 0.19 \, \text{m}$; (e) $L_x = 0.20 \, \text{m}$, $L_y = 0.20 \, \text{m}$; (f) $L_x = 0.20 \, \text{m}$, $L_y = 0.19 \, \text{m}$; (g) $L_x = 0.21 \, \text{m}$, $L_y = 0.20 \, \text{m}$; (h) $L_x = 0.21 \, \text{m}$, $L_y = 0.20 \, \text{m}$; (i) $L_x = 0.21 \, \text{m}$, $L_y = 0.19 \, \text{m}$.

Figure 5 shows the reconstruction of the crack inside the slab for all the considered models. We point out that the no–sampling approach is much more faster than the traditional sampling. More precisely, in the considered cases, once the background Green’s function has been computed, the nLSM is about 500 times faster than the classical Linear Sampling Method.

4. Conclusion

This paper presents an efficient technique for the detection of unknown scatterers inside an inhomogeneous background. The approach is based on a generalization of the linear sampling method to near-field data, cylindrical input waves and inhomogeneous background. The method has been tested in the case of synthetic data corresponding to two different two–dimensional experimental frameworks: the detection of breast tumors in a microwave tomography setting and non-destructive testing for dielectric slabs. A no–sampling implementation of the method has allowed a notable computational effectiveness, without deteriorating the visualization accuracy. In order to test the reliability of the method when the physical or geometrical parameters of the background are only approximately known, we have computed the visualizations by using Green’s functions with perturbed values of such parameters. The results show the following behaviors of the method:

- In the case of breast cancer detection, the accuracy with which the inhomogeneities are localized is rather high in all cases, even when the background Green’s functions are heavily perturbed.
• In the case of non-destructive testing, an inexact knowledge of the slab dimensions notably deteriorates the visualization of the inhomogeneity in the slab itself.
• In the case of breast cancer detection, an inexact knowledge of the fat parameters decreases the reconstruction accuracy more than an inexact knowledge of the skin parameters. This is in accordance with the previous item, since of course the fat occupies more space than the skin in the breast.

Future perspectives in this applied research activity are concerned with the extension of this analysis to three–dimensional situations and a theoretical analysis, in special cases, of the how the incomplete knowledge of the Green’s function reflects on the reconstruction performed by linear sampling.

REFERENCES


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