Infravacuum representations and velocity superselection in QED

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(joint work with Daniela Cadamuro[1])

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Sectors

1. $\mathfrak{A}$ - C^*-algebra.
2. $P_{\mathfrak{A}}$ - pure states.
3. $\mathrm{In}\,\mathfrak{A} \subset \mathrm{Aut}\,\mathfrak{A}$ - inner automorphisms.
4. $X := P_{\mathfrak{A}}/\mathrm{In}\,\mathfrak{A}$ - sectors.

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Question: Can this be done without locality?

(Second) conjugate classes

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2. For $x \in X$ set $G^a_{x,x_0} := \{\, g \in G \,|\, x = x_0 \cdot a \cdot g \,\}$.
3. $\overline{[x]}^a := \{\, x_0 \cdot a \cdot g^{-1} \,|\, g \in G^a_{x,x_0} \,\}$ is called the conjugate class.
4. $\overline{\overline{[x]}}^a := \{\, x_0 \cdot a \cdot (g')^{-1} \,|\, g' \in G^a_{y,x_0},\ y \in \overline{[x]}^a \,\}$ is called the second conjugate class.

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Claim: second conjugate classes are meaningful candidates for 'charge classes' in the absence of locality.

(Second) conjugate classes as orbits

- Def. $[x]_H := \{ x \cdot h \mid h \in H \}$ denotes the orbit.
- Def. $G_x := \{ g \in G \mid x \cdot g = x \}$ denotes the stabilizer group.

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Proposition

For $x = x_0 \cdot g_x$ we have

$$\overline{[x]}^{a} = [x_0 \cdot a \cdot g_x^{-1} \cdot a]_{G_{x_0 \cdot a}} \quad \text{and} \quad \overline{\overline{[x]}}^{a} = [x]_{G_{x_0 \cdot a}}.$$

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Background is important: For trivial background e.g. $a := e$ second conjugate classes are sensitive to any 'perturbation' of the vacuum:

$$x_0 \neq x_0 \cdot g \quad \Leftrightarrow \quad \overline{\overline{[x_0]}}^{e} \neq \overline{\overline{[x_0 \cdot g]}}^{e}.$$

Main general result

Theorem (Cadamuro-W.D. 18)

Let $R \subset S \subset G$ be subgroups. Suppose that

1. $x_0 \cdot r = x_0$ *for all* $r \in R$.
2. $x_0 \cdot s \neq x_0$ *may hold for some* $s \in S$.
3. $a \cdot S \cdot a^{-1} \subset R$.

Then, $\overline{[x_0 \cdot s]}^a = \overline{[x_0]}^a$ *and* $\overline{\overline{[x_0 \cdot s]}}^a = \overline{\overline{[x_0]}}^a$ *for all* $s \in S$.

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Definition

The relative normalizer of $R \subset S \subset G$ is defined as

$$N_G(R, S) := \{ g \in G \,|\, g \cdot S \cdot g^{-1} \subset R \}.$$

Geometric meaning of relative normalizer

1. Let $R \subset S \subset G$ and $N_G(R,S) := \{ g \in G \,|\, g \cdot S \cdot g^{-1} \subset R \}$.
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Lemma

Suppose that $x_0 \in X$ and $a \in N_G(R, S)$. Then

$$x_0 \cdot R = x_0 \quad \Rightarrow \quad (x_0 \cdot a) \cdot S = (x_0 \cdot a).$$

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Proof. $(x_0 \cdot a) \cdot s = x_0 \cdot \underbrace{(a \cdot s \cdot a^{-1})}_{r} \cdot a = x_0 \cdot a.$ □

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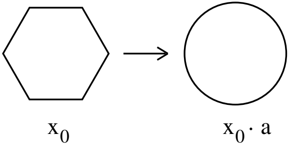

Existence of relative normalizers for $R \subsetneq S$

1. 'Tension': $R \subsetneq S$ vs $N_G(R,S) := \{\, g \in G \,|\, g \cdot S \cdot g^{-1} \subset R \,\}$.
2. Hence relative normalizers are empty for
   - abelian groups,
   - finite groups,
   - finite-dimensional Lie groups (under some assumptions).
3. However, we show that $\mathrm{ISp}(\mathcal{L})$ over an infinite dim. space $\mathcal{L}$ admits non-empty relative normalizers.
4. Their elements are Kraus-Polley-Reents symplectic maps $\hat{T}$, known as infravacua.
5. Also the resulting Bogolubov transformations $\alpha_{\hat{T}}$ are elements of relative normalizers in $\mathrm{Aut}(\mathfrak{A})$, where $\mathfrak{A} = \mathrm{CCR}(\mathcal{L})$.

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Inhomogeneous symplectic group

Symplectic group:

1. $\mathfrak{h} := L^2_{\mathrm{tr}}(\mathbb{R}^3; \mathbb{C}^3)$ - single-photon space.
2. $\mathfrak{h}_\varepsilon := \{\mathsf{f} \in \mathfrak{h} \,|\, \mathsf{f}(\mathbf{k}) = 0 \text{ for } |\mathbf{k}| \leq \varepsilon\}$
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1. $\mathcal{L}^*$ - algebraic dual. We write $\boldsymbol{v}(\mathbf{f}) = (\boldsymbol{v}, \mathbf{f})$ for $\boldsymbol{v} \in \mathcal{L}^*$, $\mathbf{f} \in \mathcal{L}$.
2. For $T : \mathcal{L} \to \mathcal{L}$ we have the transposition $T^t : \mathcal{L}^* \to \mathcal{L}^*$.
3. $\mathrm{ISp}(\mathcal{L}) := \mathcal{L}^* \rtimes_\varphi \mathrm{Sp}(\mathcal{L})$, where $\varphi(T) := (T^{-1})^t$.

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$T \in \mathrm{Sp}(\mathcal{L})$ belongs to $N_{\mathrm{ISp}(\mathcal{L})}(\mathcal{L}_{\mathrm{R}}^*, \mathcal{L}_{\mathrm{S}}^*)$ iff $(T^{-1})^t\, \mathcal{L}_{\mathrm{S}}^* \subset \mathcal{L}_{\mathrm{R}}^*$.

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Kraus-Polley-Reents infravacuum maps $\hat{T}$ satisfy this condition.

Kraus-Polley-Reents infravacuum map

1. Let $\varepsilon_i := 2^{-(i-1)}\kappa$ and $b_i := \frac{1}{i}$ for $i \in \mathbb{N}$.
2. Let $\xi_i(|\mathbf{k}|) = \frac{\chi_{[\varepsilon_{i+1},\varepsilon_i]}(|\mathbf{k}|)}{|\mathbf{k}|^{3/2}} \in L^2(\mathbb{R}_+, |\mathbf{k}|^2 d|\mathbf{k}|)$.
3. Define orthogonal projections on $\mathfrak{h} = L^2_{\mathrm{tr}}(\mathbb{R}^3;\mathbb{C}^3)$:

$$\boldsymbol{Q}_i := \frac{|\xi_i\rangle\langle\xi_i|}{\langle\xi_i|\xi_i\rangle} \otimes \sum_{0\leq\ell\leq i}\sum_{m=-\ell}^{\ell}\sum_{\lambda=\pm} |\boldsymbol{Y}_{\ell m\lambda}\rangle\langle\boldsymbol{Y}_{\ell m\lambda}|$$

4. Set $\hat{T}\mathbf{f} = \hat{T}_1(\mathrm{Re}\,\mathbf{f}) + i\,\hat{T}_2(\mathrm{Im}\,\mathbf{f})$, where $\mathbf{f} \in \mathcal{L}$ and

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Relative normalizers in $\mathrm{Aut}(\mathfrak{A})$

1. Let $\mathfrak{A}$ be the C^*-algebra generated by symbols $\{W(\mathbf{f})\}_{\mathbf{f}\in\mathcal{L}}$ s.t.

$$W(\mathbf{f}_1)W(\mathbf{f}_2) = e^{-i\sigma(\mathbf{f}_1,\mathbf{f}_2)}W(\mathbf{f}_1+\mathbf{f}_2), \quad W(\mathbf{f})^* = W(-\mathbf{f}).$$

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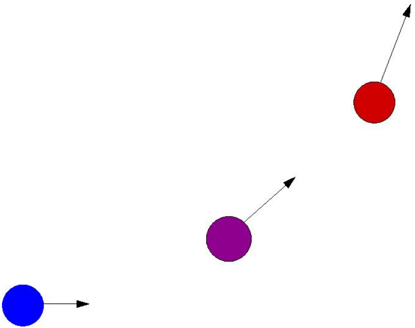
In particular $\alpha_{\hat{T}} \in N_G(R,S)$, where $\hat{T}$ is the KPR map.

Problem of velocity superselection



Free electromagnetic field

1. Single-photon space: $\mathfrak{h} := \{ \mathbf{f} \in L^2(\mathbb{R}^3; \mathbb{C}^3) \,|\, \boldsymbol{k} \cdot \boldsymbol{f}(\boldsymbol{k}) = 0 \, \}$.
2. Fock space of multi-photon states: $\mathcal{F}_{\mathrm{ph}} := \Gamma(\mathfrak{h})$.
3. Energy-momentum operators of photons:

$$H_{\mathrm{ph}} = \sum_{\lambda=\pm} \int d^3k \, |\mathbf{k}| \, a^*_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}), \quad \boldsymbol{P}_{\mathrm{ph}} = \sum_{\lambda=\pm} \int d^3k \, \mathbf{k} \, a^*_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}).$$

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Model of non-relativistic QED

1. Hilbert space: $\mathcal{H} = L^2(\mathbb{R}^3) \otimes \mathcal{F}_{\mathrm{ph}}$.
2. Energy-momentum operators:

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$$\hat{\boldsymbol{P}} = -i\nabla_{\boldsymbol{x}} \otimes 1 + 1 \otimes \boldsymbol{P}_{\mathrm{ph}}.$$

3. Fiber decomposition: $H_\sigma = I^*\big(\int^{\oplus} d^3P \, H_{\boldsymbol{P},\sigma}\big) I$, where

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Spectrum of the fiber Hamiltonians $H_{P,\sigma}$

Let $\mathcal{S} = \{\boldsymbol{P} \in \mathbb{R}^3 \,|\, |\boldsymbol{P}| < \frac{1}{3}\}$ and $\tilde{\alpha}$ small.

Lemma (Hasler-Herbst 08, Chen-Fröhlich-Pizzo 09)

Let $E_{\boldsymbol{P},\sigma} := \inf \operatorname{sp}(H_{\boldsymbol{P},\sigma>0})$ *and* $E_{\boldsymbol{P}} := \inf \operatorname{sp}(H_{\boldsymbol{P},\sigma=0})$.

1. $E_{\boldsymbol{P},\sigma}$ *is a (simple) eigenvalue with eigenvector* $\Psi_{\boldsymbol{P},\sigma}$.
2. $\mathrm{w} - \lim_{\sigma\to 0} \Psi_{\boldsymbol{P},\sigma} = 0$ *and* $E_{\boldsymbol{P}}$ *is not an eigenvalue.*
3. $\lim_{\sigma\to 0} W_0(v_{\boldsymbol{P},\sigma})\Psi_{\boldsymbol{P},\sigma}$ *exists and is non-zero, where*

$$W_0(v_{\boldsymbol{P},\sigma}) := e^{a^*(v_{\boldsymbol{P},\sigma}) - a(v_{\boldsymbol{P},\sigma})},$$

$$v_{\boldsymbol{P},\sigma}(\boldsymbol{k}) := \left(\frac{\tilde{\alpha}}{2(2\pi)^3}\right)^{\frac{1}{2}} \frac{\chi_{[\sigma,\kappa]}(|\boldsymbol{k}|)}{|\boldsymbol{k}|^{3/2}} P_{tr} \frac{\nabla E_{\boldsymbol{P},\sigma}}{1 - \hat{\boldsymbol{k}}\cdot\nabla E_{\boldsymbol{P},\sigma}}.$$

Spectrum of the fiber Hamiltonians $H_{P,\sigma}$

Let $\mathcal{S} = \{\boldsymbol{P} \in \mathbb{R}^3 \,|\, |\boldsymbol{P}| < \frac{1}{3}\}$ and $\tilde{\alpha}$ small.

Lemma (Hasler-Herbst 08, Chen-Fröhlich-Pizzo 09)

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Velocity superselection

Thm (Fröhlich 74, Chen-Fröhlich-Pizzo 09, Könenberg-Matte 14)

For any $\boldsymbol{P} \in \mathcal{S}$ the following limits exist and define states on $\mathfrak{A}$

$$\omega_{\boldsymbol{P}}(A) := \lim_{\sigma \to 0} \langle \Psi_{\boldsymbol{P},\sigma}, \pi_0(A) \Psi_{\boldsymbol{P},\sigma} \rangle, \qquad A \in \mathfrak{A}.$$

The corresponding sectors are mutually disjoint i.e.

$$[\omega_{\boldsymbol{P}_1}]_{\mathrm{In}\mathfrak{A}} \neq [\omega_{\boldsymbol{P}_2}]_{\mathrm{In}\mathfrak{A}} \quad \text{for} \quad \boldsymbol{P}_1 \neq \boldsymbol{P}_2.$$

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Theorem (Cadamuro-W.D. 18)

Let $\hat{T}$ *be the KPR infravacuum. Then, for all* $\boldsymbol{P}_1, \boldsymbol{P}_2 \in \mathcal{S}$

$$\overline{[[\omega_{\boldsymbol{P}_1}]_{\mathrm{In}\mathfrak{A}}]}^{\alpha_{\hat{T}}} = \overline{[[\omega_{\boldsymbol{P}_2}]_{\mathrm{In}\mathfrak{A}}]}^{\alpha_{\hat{T}}}, \quad \text{and} \quad \overline{\overline{[[\omega_{\boldsymbol{P}_1}]_{\mathrm{In}\mathfrak{A}}]}}^{\alpha_{\hat{T}}} = \overline{\overline{[[\omega_{\boldsymbol{P}_2}]_{\mathrm{In}\mathfrak{A}}]}}^{\alpha_{\hat{T}}}.$$

Outlook

1. We exhibited a mathematical structure underlying the infravacuum: the relative normalizer of $R \subset S \subset G$:

$$N_G(R, S) := \{ g \in G \,|\, g \cdot S \cdot g^{-1} \subset R \}.$$

2. We propose the second-conjugate class w.r.t. the infravacuum background as a 'charge class' collecting sectors differing by unobservable 'soft-photon clouds'.

3. Question: How large are the second-conjugate classes?

4. Question: How to incorporate positivity of energy?

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