Quantum-mechanical backflow and scattering

Gandalf Lechner

joint work with Henning Bostelmann and Daniela Cadamuro

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History:

- **1969:** First description of the effect [Allcock]
- **1998:** First quantitative analysis of backflow [Bracken/Melloy]
- **2003:** Backflow as a quantum inequality [Eveson/Fewster/Verch]
- **2013:** Suggestion for experimental observation [Palmero et. al.]

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- In this talk, only non-relativistic quantum mechanics

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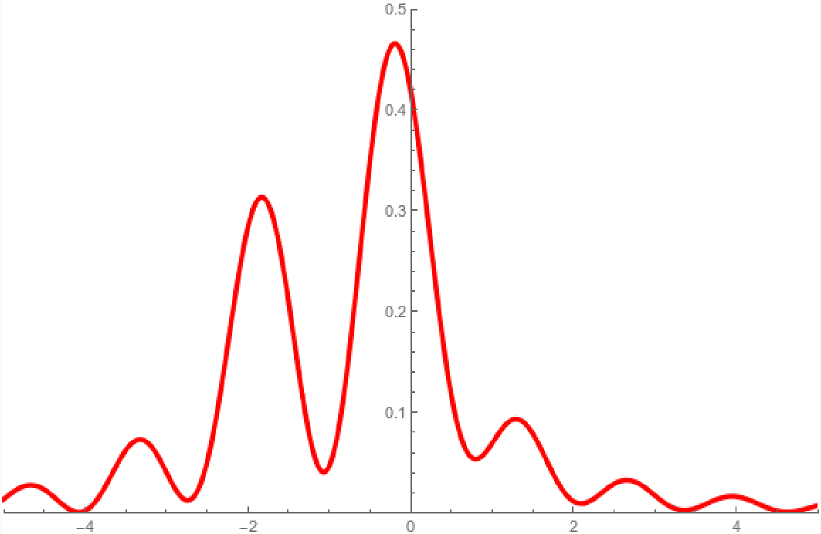

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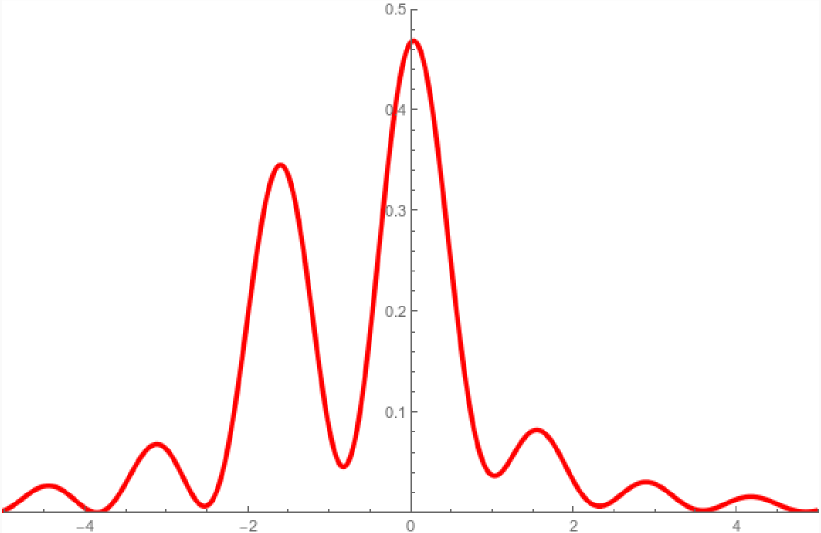

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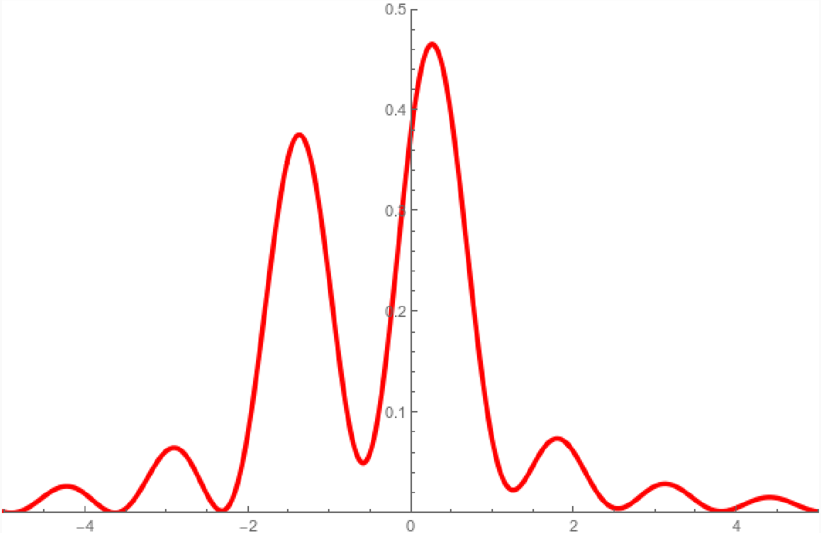

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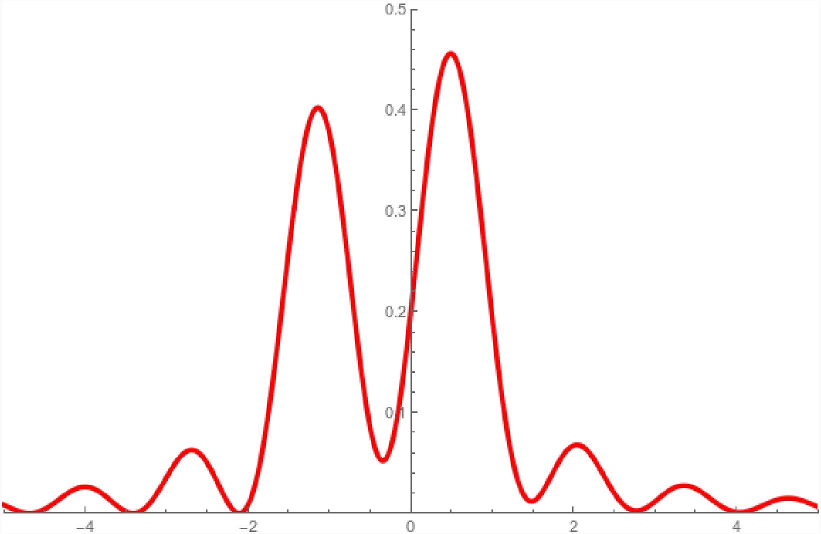

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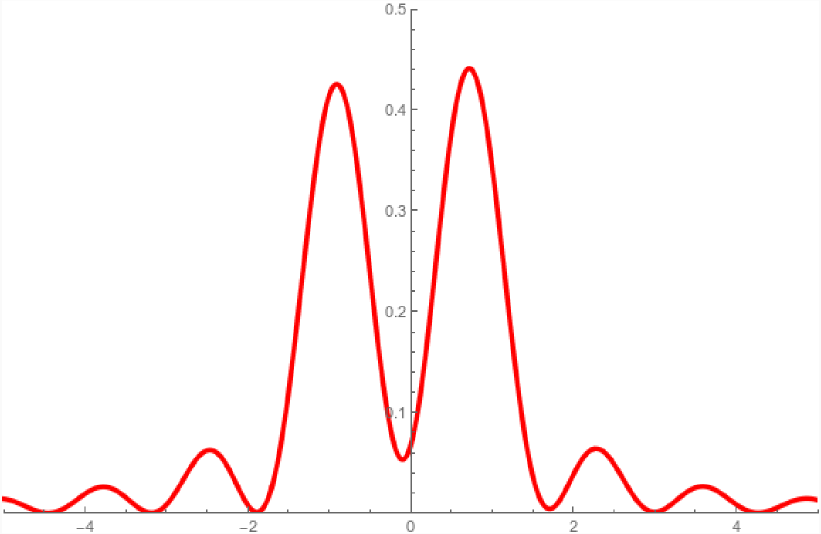

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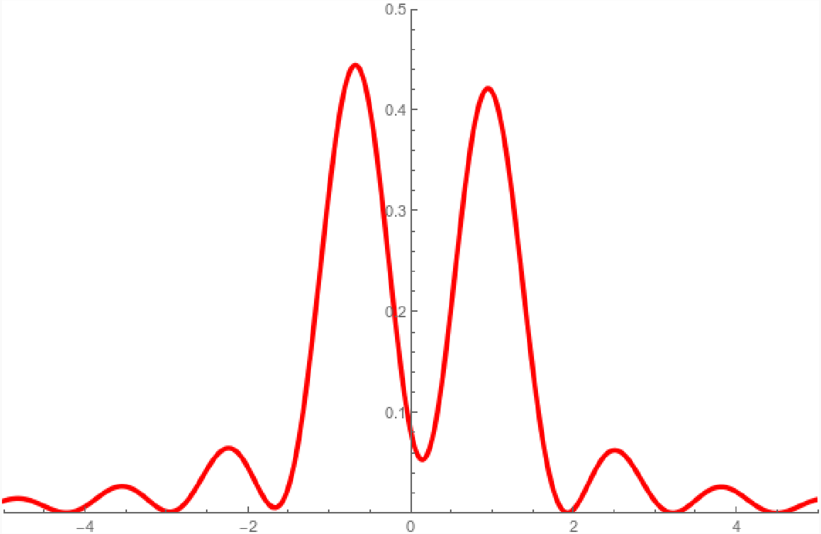

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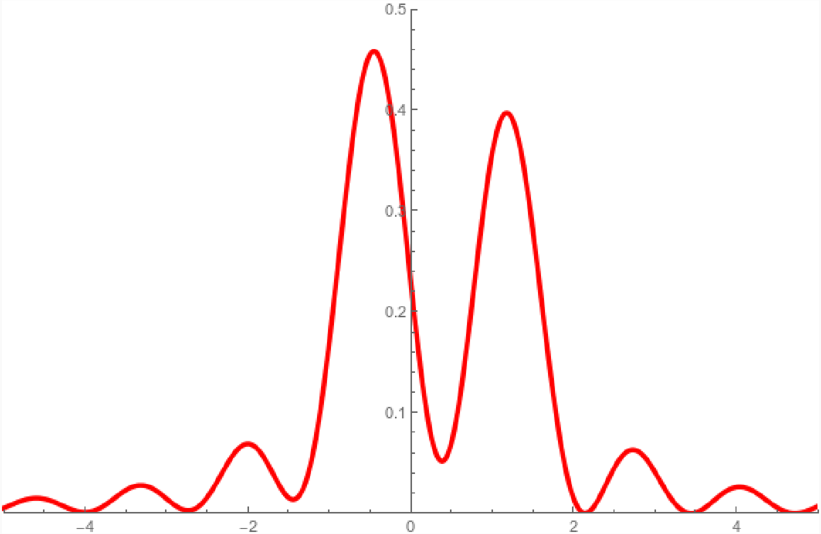

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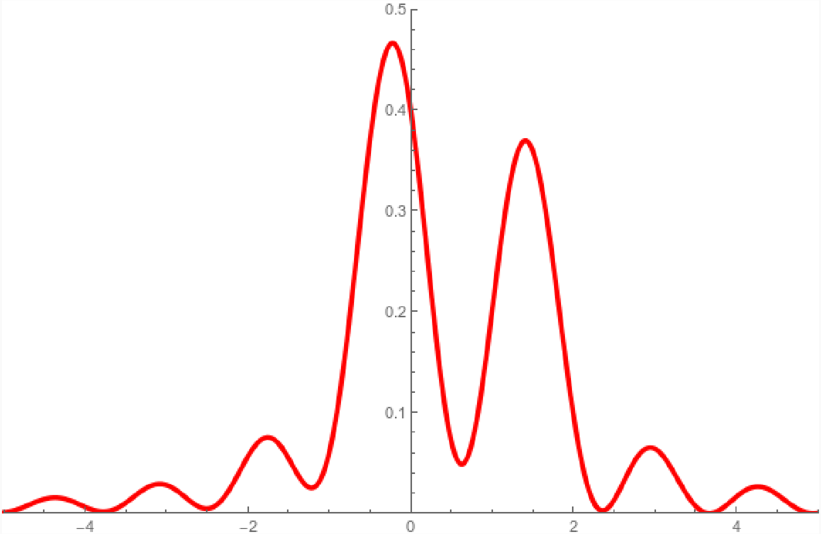

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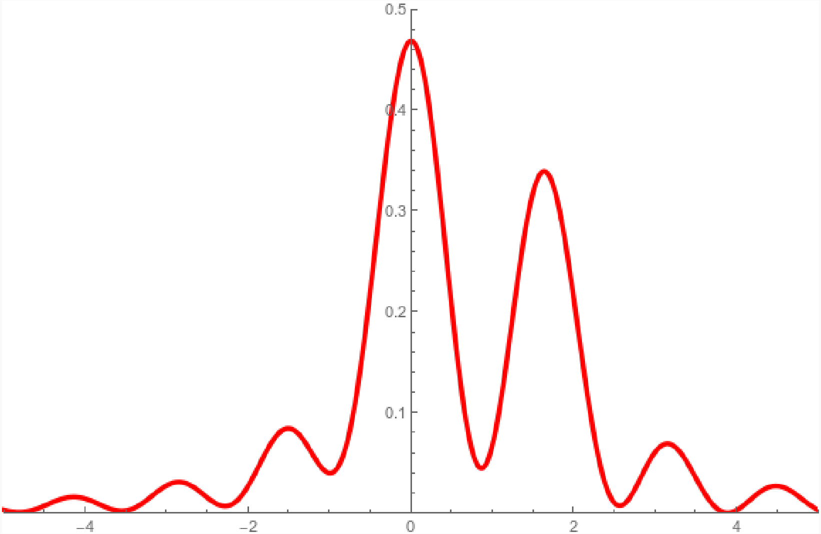

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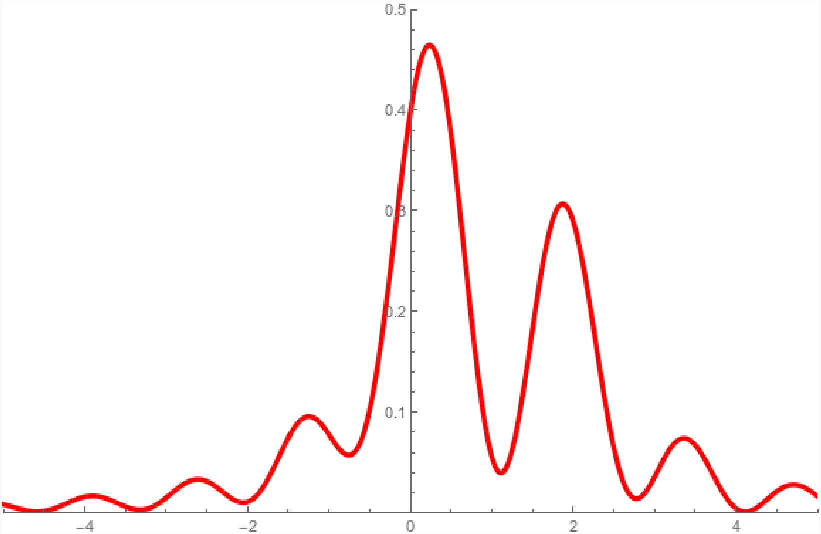

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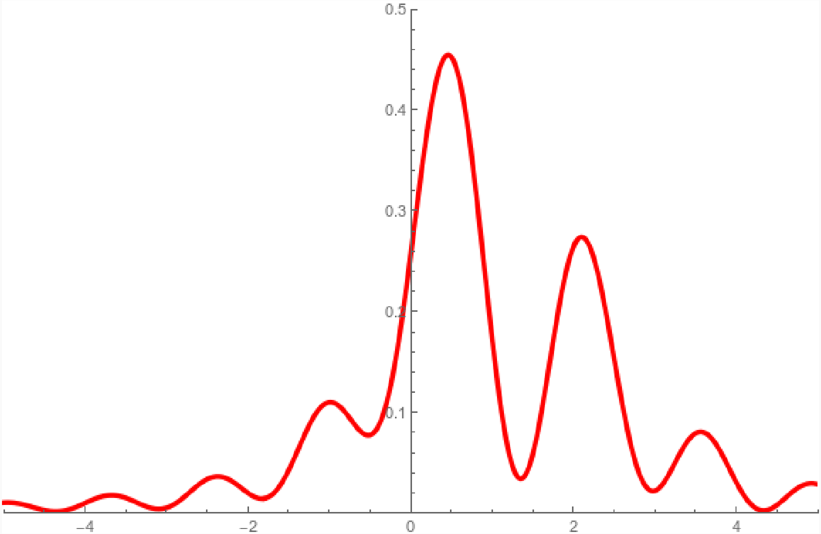

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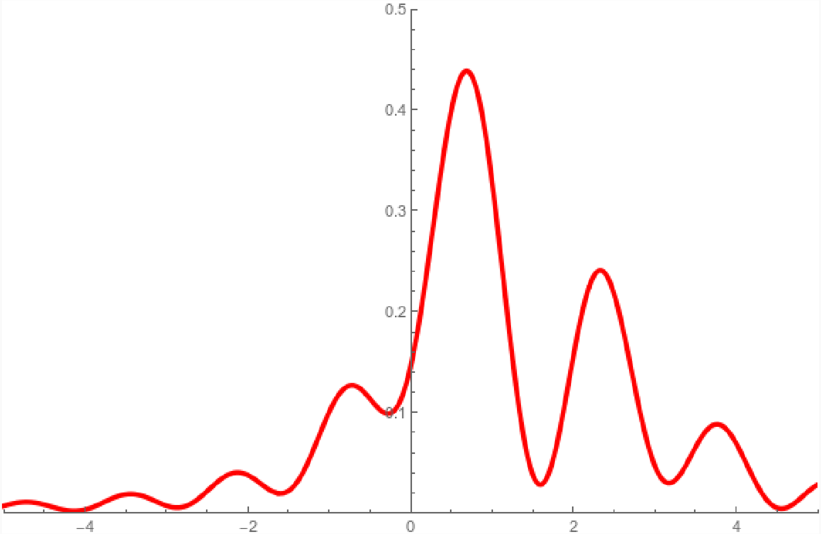

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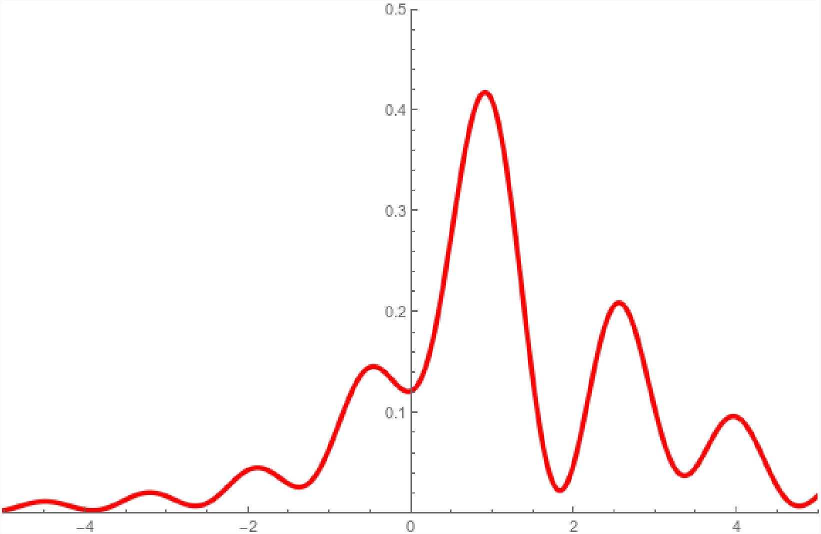

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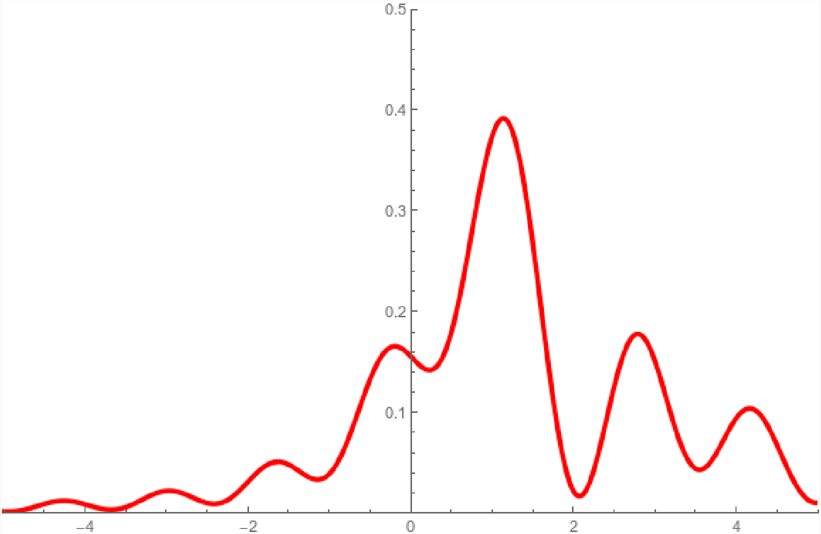

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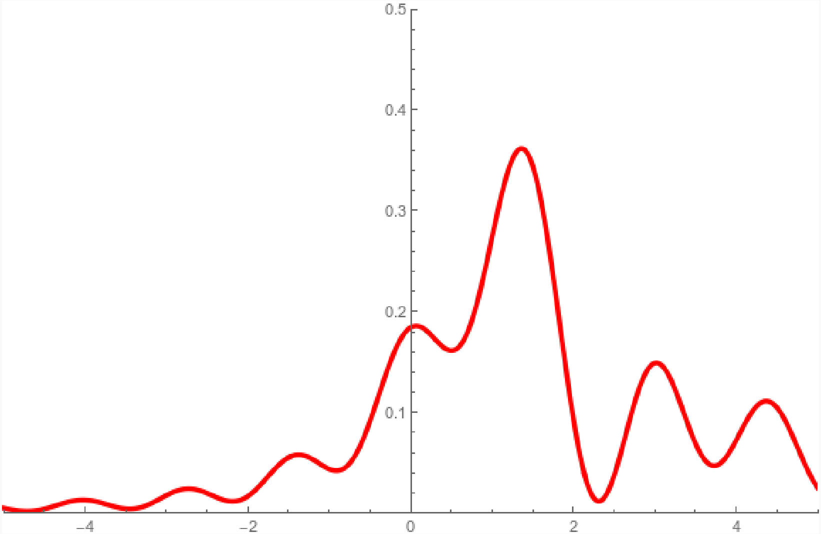

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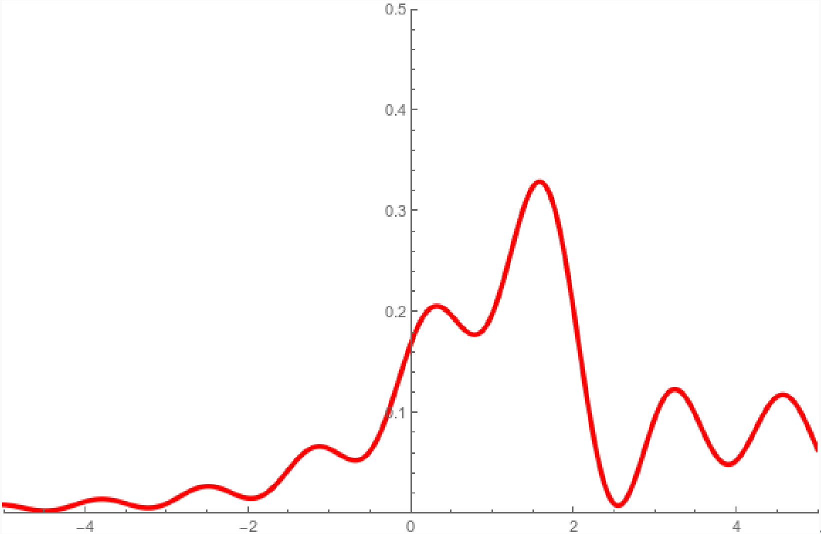

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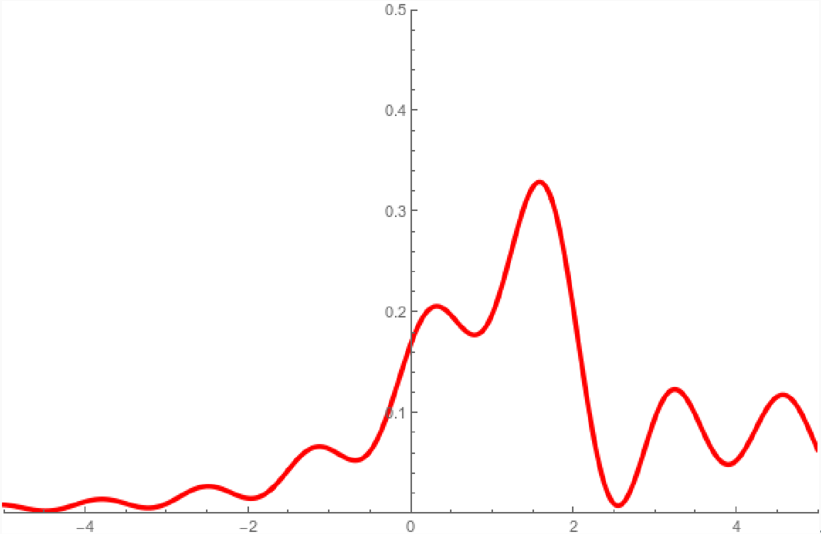

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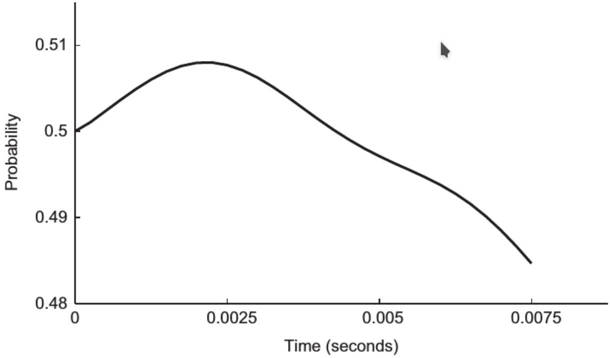

Fig. 10. Probability that the quantum bus is to the left of $x=0$, versus time in seconds.

In "bus picture", this means that the probability $L(t)$ that the bus has not passed already can **increase** with waiting time t!

Mathematical setup

- Normalized wave function $\psi \in L^2(\mathbb{R})$ has probability density $|\psi(x)|^2$ (for position) and probability current density

$$j_\psi(x) = \frac{i}{2}\left(\overline{\psi'(x)}\psi(x) - \overline{\psi(x)}\psi'(x)\right).$$

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- j_ψ is purely kinematical. When dynamics with Hamiltonian H is fixed, write $j_{e^{-itH}\psi}(x) =: j_\psi^H(t,x)$.
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Backflow is the fact that $a) \not\Longrightarrow b)$**, where**

a) ψ contains only positive momenta, i.e. $\operatorname{supp}\tilde{\psi} \subset \mathbb{R}_+$

b) $j_\psi(x) > 0$ for all $x \in \mathbb{R}$

- Current quantum field (quadratic form)

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Lemma

For any $x \in \mathbb{R}$, the quadratic form $E_+ J(x) E_+$ is unbounded above and below.

- Unboundedness above: high momentum effect.
- Unboundedness below: take "backflow states" as superposition of high and low (positive) momentum.

Bounds on backflow

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- Amount of probability flowing across $x = 0$ in time interval $[0, T]$:

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- Examples of “quantum inequalities” [Fewster]
- Known results are either about interaction-free situation, or of purely kinematical nature.

Backflow and interaction (scattering)

- What can we say about backflow in an **interacting** system?
- For example, take Hamiltonian $H = \frac{1}{2}P^2 + V(X)$ with potential V.
- For non-constant V, the space $E_+\mathcal{H}$ of right-movers is not preserved by time evolution e^{-itH}.
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- Reflection = "classical backflow" (amplifies backflow). Is this still a "small effect"?
- Interesting special case: Reflection-less potentials as non-relativistic analogues of integrable QFTs. Do they have backflow?

Potential scattering in quantum mechanics

- Time-dependent setting: $H = \frac{1}{2}P^2 + V(X)$ full Hamiltonian, $H_0 = \frac{1}{2}P^2$ free Hamiltonian. **Møller operator**:

$$\Omega_V = \operatorname*{s-lim}_{t\to-\infty} e^{itH} e^{-itH_0} \, ,$$

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Theorem

Let $V \in L^1(\mathbb{R})$ satisfy $\int_{\mathbb{R}} dx\,(1+|x|)|V(x)| < \infty$ ("short range"). Then

a) Ω_V exists.

b) $-\frac{1}{2}\partial_x^2\varphi_k(x) + V(x)\varphi_k(x) = k^2\,\varphi_k(x)$, $k > 0$, has unique solution

$$\varphi_k(x) = \begin{cases} e^{ikx} + R_V(k)e^{-ikx} + o(1) & x \ll 0 \\ T_V(k)e^{ikx} + o(1) & x \gg 0 \end{cases}$$

c) For $\tilde{\psi} \in C_0^\infty(\mathbb{R}_+)$, have $(\Omega\psi)(x) = \frac{1}{\sqrt{2\pi}}\int_0^\infty dk\,\varphi_k(x)\tilde{\psi}(k)$.

Backflow and scattering

The backflow in a scattering situation is measured by the operator, $f \geq 0$,

$$\langle \psi, E_+ \Omega_V^* J(f) \Omega_V E_+ \psi \rangle = \int dx\, f(x)\, j_{\Omega_V E_+ \psi}(x)\,.$$

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Given arbitrary $V \in L^{1+}(\mathbb{R})$ and arbitrary $x \in \mathbb{R}$, the quadratic form $E_+ \Omega_V^* J(x) \Omega_V E_+$ is unbounded above and below.

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Idea of proof: Comparison with free case ("perturbation theory"). Relies on known estimates on the difference between $\varphi_k(x)$ and its asymptotics (e.g. [Deift/Trubowitz 79]).

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To show that the averaged current $E_+\Omega^* J(f)\Omega E_+$ is bounded below, expand

$$\begin{aligned} E_+\Omega^* J(f)\Omega E_+ = {} & E_+\Omega^* E_+ J(f) E_+ \Omega E_+ \\ & + E_+\Omega^* E_+ J(f)(i+P)^{-1} E_-(i+P)(\Omega-1)E_+ \\ & + E_+(\Omega^*-1)(i+P)E_-(i+P)^{-1}J(f)\Omega E_+ . \end{aligned}$$

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- We have $\|J(f)(i+P)^{-1}\| < \infty$
- Relevant term to estimate is $P(\Omega-1)$.

Mathematical question (work in progress)

Let H_0, H be selfadjoint on a Hilbert space $\mathcal{H}$, such that Møller operator Ω exists. For which (unbounded) functions g (e.g. $g(\lambda) = \lambda^\alpha$, $g(\lambda) = e^\lambda$...) do we have

$$\|g(H_0)(\Omega - 1)\| < \infty \text{ ?}$$

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Examples:

- $\mathcal{H} = L^2(\mathbb{R})$, $H_0 = P$, $H = P + V(X)$ with $V \in C_0^\infty(\mathbb{R})$. Then $(1 + H_0^2)^\varepsilon(\Omega - 1)$ is **unbounded** for every $\varepsilon > 0$.

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- $\mathcal{H} = L^2(\mathbb{R}^n)$, $H_0 = -\Delta$, $H = H_0 +$ **integral operator** with kernel $K \in \mathcal{S}(\mathbb{R}^n \times \mathbb{R}^n)$ and supp $\tilde{K}$ **compact**. Then $\|g(H_0)(\Omega - 1)\| < \infty$ for all functions g. (similar to [GL/Verch 15])

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In general – between these two extremes – a lot of relevant information seems to be encoded in the boundary behavior of the resolvents of the Hamiltonians (LAP).

Backflow and scattering

In concrete 1d QM case (i.e. $\mathcal{H} = L^2(\mathbb{R})$, $H_0 = \frac{1}{2}P^2$, $H = H_0 + V(X)$), use integral form of Schrödinger eqn (Lippmann-Schwinger),

$$\varphi_k(x) = T_V(k)e^{ikx} + \int dy\, V(y)\, G_k(x-y)\, \varphi_k(y)$$

and estimate

$$(P(\Omega - T_V)E_+\psi)(x) = \frac{1}{\sqrt{2\pi}i}\frac{d}{dx}\int_0^\infty dk \int_{\mathbb{R}} dy\, V(y) G_k(x-y)\varphi_k(y)\tilde{\psi}(k)$$

using known asymptotics of the solutions $\varphi_k(x)$

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Let $V \in L^{1+}(\mathbb{R})$ and $f \geq 0$. Then $E_+\Omega_V^* J(f)\Omega_V E_+$ is bounded below.

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$$(P(\Omega - T_V)E_+\psi)(x) = \frac{1}{\sqrt{2\pi} i}\frac{d}{dx}\int_0^\infty dk \int_{\mathbb{R}} dy\, V(y) G_k(x-y)\varphi_k(y)\tilde{\psi}(k)$$

using known asymptotics of the solutions $\varphi_k(x)$

Theorem: Backflow is bounded in any (short-range) potential

Let $V \in L^{1+}(\mathbb{R})$ and $f \geq 0$. Then $E_+\Omega_V^* J(f)\Omega_V E_+$ is bounded below.

- Reflection processes do not destroy boundedness of backflow.
- Heuristic explanation: Unboundedness below could only occur at high momentum, but for high momentum, reflection processes are supressed sufficiently well.

Examples and Numerics

Numerical results on specific potentials:

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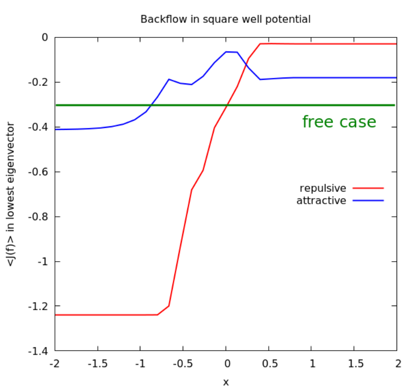
- Textbook example $V(x)$ =square well potential.



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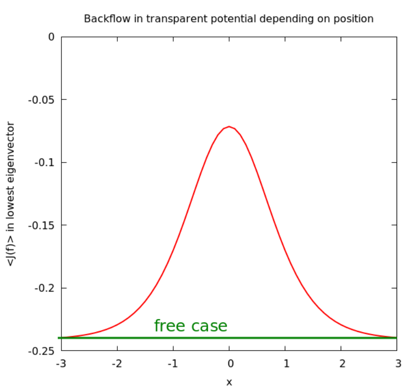
- Transparent Pöschl-Teller potential $V(x) = -\frac{\ell(\ell+1)}{2\cosh^2 x}$, $\ell \in \mathbb{N}$.



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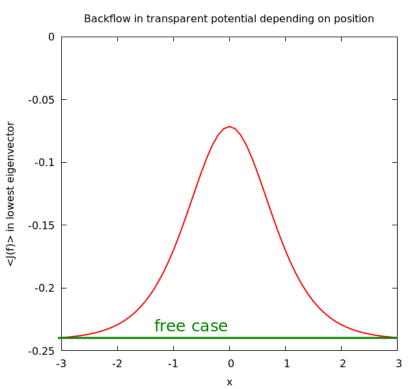

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Transparent potentials and integrable QFT

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Question

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Conclusion/Outlook

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- Generalizations: Higher dimensions, multi-particle systems, other PDEs, quantum field theory, … require general analysis of bounds on Møller operators.
- Transparent potentials have special “backflow profiles” (left and right asympotics “free”)
- As a QFT analogue, can integrability of a QFT be characterized in terms of a quantum inequality?