

Logic hidden in Mathematics

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Hermann Weyl is credited with the statement that ‘Logic is the hygiene which the mathematician practices to keep his ideas healthy and strong’ and André Weil with the extension ‘Mais si la logique est l’hygiene du mathématicien, ce n’est pas elle qui lui fournit sa nourriture’ Such views fail to recognise the the many ways in which sensitivity to logical issues can support the understanding of mathematics. One important aspect concerns the relation between abstract and concrete proofs. An example of thinking along such lines can be detected already in Euclid’s treatment of the Pythagoras Theorem. I shall explain that and go on to consider simple examples from group theory and non-commutative algebra. Then I shall discuss linear algebra and give (Weyl’s!) Principle of the Irrelevance of Algebraic Inequalities as an example of logic in action. Finally I shall describe some simple examples of categorical logic arising in the foundations of algebraic geometry.