

The Taylor formula of implied volatility

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In a model driven by a multi-dimensional local diffusion, we study the behaviour of implied volatility IV and its derivatives with respect to log-strike k and maturity T near expiry and at the money. We recover explicit limits of the derivatives of the IV for $(T, x - k)$ approaching the origin within the parabolic region $|x - k| \leq \lambda\sqrt{T}$, with x denoting the spot log-price of the underlying asset and where λ is a positive and arbitrarily large constant. Such limits yield the exact Taylor formula for implied volatility within the parabola $|x - k| \leq \lambda\sqrt{T}$. In order to include important models of interest in mathematical finance, e.g. Heston, CEV, SABR, the analysis is carried out under the weak assumption that the infinitesimal generator of the diffusion is only locally elliptic.