A Bankruptcy Approach to Ambulances Location Problem

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Abstract
The location of the emergency units may have a deep influence on the intervention time, that in its turn may be relevant for a positive solution of an emergency situation. In this paper we analyze a particular location problem with a discontinuous intervention time function. The problem is tackled from different points of view each one resulting in a different solution. The test refer to a toy model and to a real world situation of locating some ambulances in the province of Milan.
Key Words
Emergency Management, Cooperative Game, Location Problem

1 Introduction

Emergency management represents a hard question in several situations. It has to be viewed under many different lights, so that a large number of parameters is involved. Consequently, many different experiences and expertizes are needed in order to have an efficient result. First of all, it is necessary to distinguish among different types of emergency: medical, environmental, urban, and so on. Each of them has particular features that have to be tackled in different ways, with different means and tools. Before going deeper in our analysis we want to devote some words to make clear the characteristics of the kind of situations that are more suitable for our approach.

The idea of emergency is strongly correlated to the idea of urgency, and then with the idea of fast intervention. On the other hand the measure of the time required is different for the various types of emergency. Medical intervention should be carried out in minutes, while environmental emergency may allow longer intervention time and the intervention after an earthquake can be organized also some days after the event.

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In this work we are interested in those particular situations for which the utility of the intervention has a discontinuity, i.e. after a given period there is a sudden and large reduction of the utility of the intervetion. A classical example is represented by the ambulances that are allowed for a maximal time for the most difficult situations, the so-called *red* and *yellow codes*; for example in Italy the time allowed is 8 minutes, including the time for answering the call. It is clear that if an ambulance arrives after 9 minutes the intervention is anyhow completed (for example an injured person is transported to the hospital), but when an ambulances is located on the territory only the area that can be reached in 8 minutes is considered as covered. Another interesting situation is represented by antifire planes or helicopters, for which it is necessary to take into account the fuel tank capacity and the distance from a place for charging water.

This paper is organized as follows: in the next section we recall some definitions and solutions of bankruptcy problems; in Section 3 we introduce the problem; Section 4 is devoted to a simple theoretical model for testing our approach; a more complex situation arising from the real world is described in Section 5; Section 6 concludes.

2 Preliminaries on Bankruptcy

A classical bankruptcy problem arises from a situation where an estate E has to be divided among a set $N = \{1, 2, ..., n\}$ of claimants, each of them with a claim $c_i,\ i = 1, \ldots, n$ on the estate, and the total claim exceeds the available estate.

Formally, a bankruptcy problem is an ordered triplet (N, E, c) where $E \in \mathbb{R}_+, c \in \mathbb{R}^n_+$ and $E \leq c_1 + c_2 + ... + c_n$.

A solution of a bankruptcy problem is a vector $x = (x_1, x_2, ..., x_n)$ such that:

$$
\begin{aligned}
&0 \leq x_i \leq c_i, \qquad \text{for each } i \in N, \\
&\sum_{i=1}^{n} x_i = E,
\end{aligned}
$$

where x_i can be interpreted as the part of the estate E assigned to claimant i.

A division rule is a function f which assigns to any bankruptcy problem (N, E, c) a vector $f(N, E, c) \in \mathbb{R}^n$ that is a solution of the bankruptcy problem.

Well known division rules are the proportional rule ($PROP$), the constrained equal award rule (CEA) and the constrained equal loss rule (CEL).

We start by briefly describing these rules:

(i) The i-th coordinate of $PROP(N, E, c)$ is given by

$$PROP_i(N, E, c) = \left(\sum_{i \in N} c_i \right)^{-1} c_i E,\ \ i \in N.$$

(ii) The i-th coordinate of $CEA(N, E, c)$ is given by

$$CEA_i(N, E, c) = \min\{c_i, \alpha\},\ i \in N,$$

where α is the unique real number so that $\sum_{i \in N} CEA_i(N, E, c) = E$.

(iii) The i-th coordinate of $CEL(N, E, c)$ is given by

$$CEL_i(N, E, c) = \max\{c_i - \beta, 0\},\ i \in N,$$

where β is the unique real number so that $\sum_{i \in N} CEL_i(N, E, c) = E$.

3 The Problem

By their definition, bankruptcy problems may be applied when we need to allocate a scarce resource which is not sufficient to cover all the requests of the agents. So, the problem of allocation of ambulances for an emergency service can be studied under this point of view, as, in general cases, we have a number of ambulances far below the requests originated by a given area.

From the point of view of *efficiency*, the aim is to cover the maximum number of calls which are received from the area we need to attend to. From the one of *equity*, the aim could be the maximum coverage of the area, without taking care of the number of calls received. An intermediate point of view could be represented by the attempt to reach as many people as possible.

In order to simplify the approach, the total area we consider is divided according to a grid. Let $m \times n$ be the dimensions of the grid. A number representing its own value is assigned to each box, whose meaning depends on the point of view we choose (see above).

These preliminar choices led to the definition of the bankruptcy problem. First of all, we need to choose an enumeration of the boxes of the grid. Let l be the number of candidate locations for an ambulance, where $a_1, \ldots, a_l$ represents their position in the considered enumeration of the boxes, with $l \leq mn$. Then, let $r_i, i = 1, \ldots, l$ be the value of the box i and k be the number of ambulances we have. We can consider two different approaches to the problem. For each of them, the number of resources E is represented by the number of ambulances k, the set of agents $N = \{1, \ldots, l\}$ is the set of possible locations, while the vector of requests c vary with the approach we consider.

The first bankruptcy problem is based on an *a priori aggregation* of the values of the boxes an ambulance can cover. Then, the set of agents will be $N = \{a_1, \ldots, a_l\}$, i.e. the set of feasible allocations for an ambulance. If S_{a_j} is the set of the boxes covered by the agent a_j, then the number of requests for it will be $c_{a_j} = \sum_{k \in S_{a_j}} r_k$ and the vector of requests will be $c = (c_{a_1}, \ldots, c_{a_l})$. Then we can solve the problem using one of the methods described before ($PROP$, CEA, CEL), finding the number of ambulances to be assigned to each location.

The second one is based on an *a posteriori aggregation*. In this case we considerfirst the problem of allocating the ambulances among all the boxes of the grid implementing one of the possible methods of solution to this bankruptcy problem. Let $b = (b_1, \ldots, b_n m)$ be the solution. Using the notation introducted before, the number of ambulances we assign to location a_j is given by $\sum_{k \in S_{a_j}} b_k$.

As in both cases the number of ambulances assigned to each location could be a non-integer number, while we are obviously interested in assigning an integer number of vehicles, we need to find a way to round this number to an integer, without affecting the feasibility of the solution. One of the possible methods could be the following one:

(i) let a_J be the location which received the largest number M of vehicles;

(ii) we assign to a_J the minimum between an integer approximation of M and the number of remaining ambulances;

(iii) if the number of remaining ambulances is larger than 0, we turn back to step (i).

In this way, we can assign all the vehicles we have to the feasible locations.

Using the three possible solution methods we described, i.e. $PROP$, CEA and CEL, both with the a priori aggregation and with the a posteriori aggregation, we obtain six solutions to the problem of ambulances allocation for an emergency service. In the following, we will indicate with an index (1) the solution obtained by the a priori aggregation, with a (2) the one obtained using the a posteriori aggregation.

A preliminary study of the problem leads to the remark that only four out of the solutions we can obtain are significant.

We may notice that the proportional solution is not influenced by the kind of the aggregation. For a player a_j, infact, we have

$$PROP^{(1)}_{a_j} = \frac{\sum_{k \in S_{a_j}} c_k}{C} E = \sum_{k \in S_{a_j}} \frac{c_k}{C} E = PROP^{(2)}_{a_j}$$

It is well known that the proportional rule is not manipolable (see ...)

Moreover, the CEA solution obtained by an a priori aggregation, in realistic cases, assignes 1 ambulance to each of the first k location. In fact, the starting α in the CEA algorithm is $\frac{E}{n} = \frac{k}{l} < 1$, as the number of ambulances is in general less than the number of feasible locations we have. On the other hand, in real-world cases, the value (calls, area, population) assigned to the area covered by a location is larger than 1. So, for each agent a_j we obtain

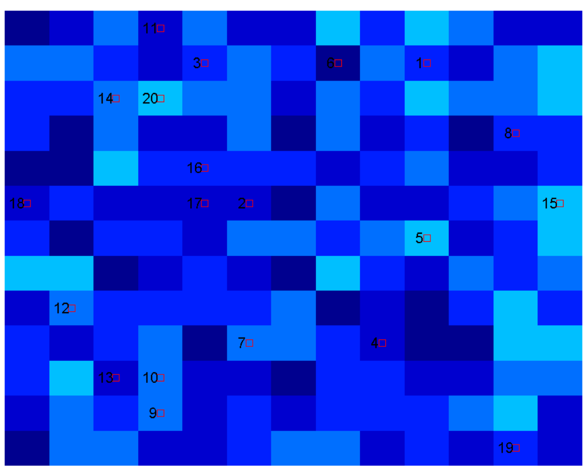
$$\begin{array}{ll} CEA_i^{(1)} = min(\frac{k}{l}, \sum_{k \in S_{a_j}} c_k) & \\ \frac{k}{l} < 1 & \implies \quad CEA_{a_j}^{(1)} = \frac{k}{l} \\ \sum_{k \in S_{a_j}} c_k > 1 & \end{array}$$

So we assign to each player the same number of vehicles $\frac{k}{l}$ and the actual ambulances assignement depends only on the order in which we check the locations. This situation does not occur in the a posteriori aggregation problem, as the boxes are considered separately and the value of each of them could be less than the value $\alpha = \frac{k}{mn}$ of this problem, resulting in different values of the solution $CEA^{(2)}$ for the different players.

4 A Case Study

Now we introduce an example which may clarify the bankruptcy approach to the emergency service allocation problem.

Figure 1: map of the example model



Let us consider a city as depicted in Figure 1. This city is represented by a 13×13 grid in which the numbered squares indicate the 20 possible locations of an ambulance in the area. The colour of each box of the grid indicates the average number of calls received from that zone (the darker the colour, the more the calls). We assume that a vehicle can cover a distance of two steps in the so called *Manhattan metric*, i.e. moves are allowed only along the vertical and the horizontal directions. Mathematically, the Manhattan metric corresponds to the metric associated to the unitary norm and it is very suitable for representing distances in urban areas, where it is necessary to follow the streets that usually cross ortogonally. Moreover, let 10 be the number of ambulances we need to allocate on the city area.

In Table 1 we summarize the results of the analysis on this example. The solutions have been implemented using a Matlab program for these types of problems, which gives the results in real time.

Table 1: results of the example

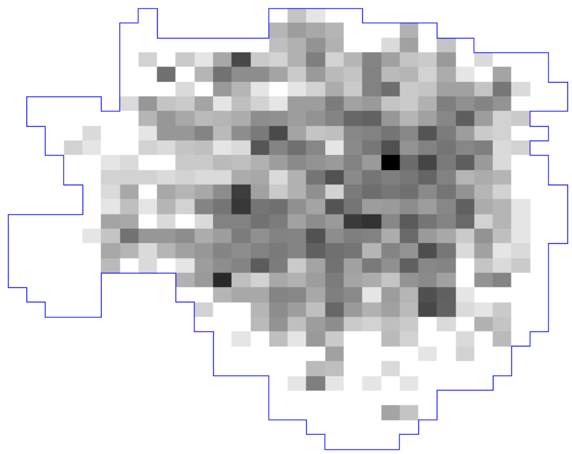
Player	$PROP^{(1)}$	$PROP^{(2)}$	$CEA^{(1)}$	$CEA^{(2)}$	$CEL^{(1)}$	$CEL^{(2)}$
1	0	0	1	0	0	0
2	1	1	1	1	0	0
3	1	1	1	0	0	1
4	1	1	1	1	4	5
5	1	1	1	1	5	4
6	1	1	1	0	0	0
7	1	1	1	1	0	0
8	0	0	1	0	0	0
9	0	0	1	0	0	0
10	1	1	1	1	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	1	0	0	0
14	0	0	1	0	0	0
15	0	0	0	0	0	0
16	1	1	1	0	0	0
17	1	1	1	0	1	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	1	1	1	0	0	0

These results show that only 4 solutions are significant, as we said before. We can also notice that the CEL solution is the only one that assigns more than an ambulance to a location. This occurs because it favours those locations with a larger number of calls.

We can also notice that only two locations (4 and 5) obtain at least 1 ambulance under each type of significant solution. These can be very good candidates to obtain a vehicle. Moreover, 10 locations (1, 8, 9, 11, 12, 13, 14, 15, 18 and 19) do not obtain any ambulance under any type of solution, so they are good candidate to receive no vehicle. This occurs because each type of solution take care of a different point of view of the problem we are analyzing: to give a proportional division, to favour the locations with less requests (CEA) or the ones with more.

5 A real case

Figure 2: map of Milan



In Figure 2 is represented a map of the urban area of Milan. The city is divided usign a grid in which each square has an edge of 0.5 km. The colour of each box represents the average number of calls received from each part of the city in the period 4 p.m.-5 p.m. (again, the darker the colour, the more the calls). We assume that each vehicle can perform a maximum of 6 steps in the Manhattan metric and that an ambulance can be located in every box. These two assumptions are suitable for the case of Milan, because in the urban area the speed of an ambulance can be approximated in 30 km/h and a parking for a vehicle can be built in each area represented by a box.

Our aim is to cover all the calls which originate from the city. We implemented a Matlab program which solves the problem with the following steps:

1. it calculates the CEA solution with a posteriori aggregation on the situation we have;

2. it associates an ambulance to the player (i.e. the box) which receives the maximum from the solution;

3. a zero is then assigned to the boxes in the part of the city covered by this player;

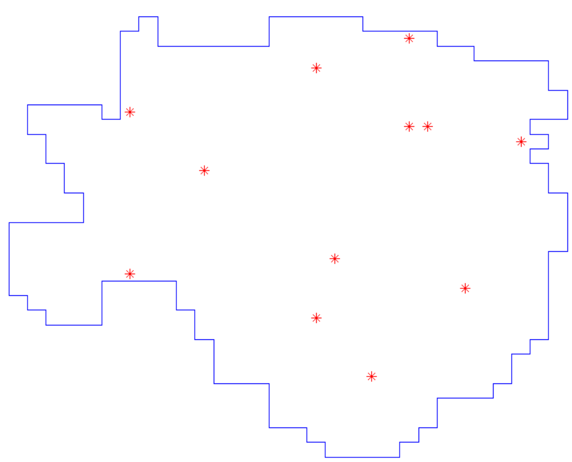
4. the program calculates the part of the city which is covered by the ambulances which are placed in the city;
5. if this part does not correspond to 100%, we return to step 1..

The results are shown in the following Table 2 and in Figure 3.

Table 2: results of the first approach to the real-world case

Ambulances	Covered Area
1	29.74
2	55.58
3	70.82
4	80.80
5	84.36
6	89.14
7	95.56
8	96.13
9	98.04
10	99.08
11	99.97
12	100.00

Figure 3: map of the results of the first approach



In this way we have assumed that an ambulance located in a certain zone of the city can attend to all the calls originated in an hour from the part of the urban area it covers. This is a very strong assumption and it does not correspond to the real case, but this solution can give a first idea of the minimal number of ambulances which is needed to cover the whole city.

In order to get partially around the problem, we propose a second solution, subsituting step 3. in the previous algorithm with

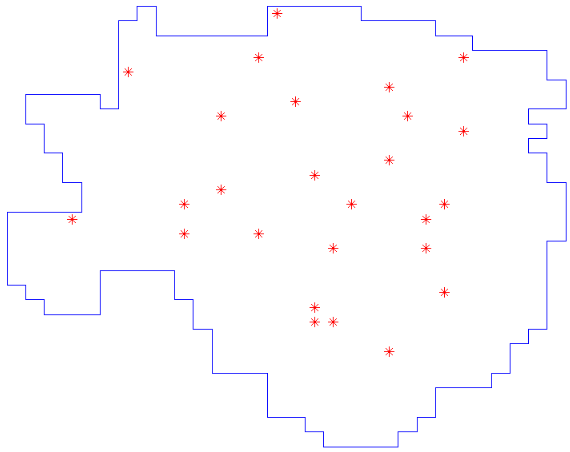
3'. the number of calls received from each box in the area covered by the located ambulance is reduced by 1;

In this way, we assume that a player cannot attend to more than one call originating from a box it covers. The results we obtain are summarized in Table 3 and in Figure 4.

Table 3: results of the second approach to the real-world case

Ambulances	Covered Area
1	15.73
2	30.40
3	41.80
4	51.63
5	61.56
6	66.33
7	71.89
8	79.48
9	82.73
10	86.45
11	87.92
12	90.70
13	92.27
14	93.59
15	94.03
16	95.52
17	95.82
18	95.91
19	97.14
20	97.90
21	98.39
22	98.67
23	98.93
24	99.13
25	99.34
26	99.54
27	99.71
28	99.80
29	100.00

Figure 4: map of the results of the second approach



In this case we need 29 ambulances instead of 12, which is a more realistic valuation of the amubulances required to cover the city of Milan. However, this algorithm assumes that an ambulance can attend up to 85 calls in an hour, which does not correspond to a realistic situation.

6 Concluding Remarks

The results obtained up to now are satisfactory and encouraging for future researches.

The results summarized in Table 2 and in Table 3 may give an idea of the number of ambulances we need to cover the urban area of Milan. Moreover, the percentage cover data can give an idea of the problem of marginalities. As we can see in Table 3, the 95% of the city area can be covered with 16 ambulances, instead of the 29 which are needed to cover the whole area. Because of the high costs of an ambulance, thiese results may lead to a cost study of the problem.

As we discuss in the previous section, we assume that an ambulance can cover a large number of calls per hour, working under an irrealistic hypothesis. So, a possible development of the study may concern this aspect, implementing a more realistic situation in the algorithm.