A partial cooperative approach to environmental problems

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Abstract: *In this paper we study an approach through partial coopertive games to environmental topics. In general no all players wish to cooperate to solve a common problem, so we consider a model where only some decisors cooperate. Starting from the trasformation of a coalition game into a strategic one, we give a new concept of solution for partial cooperative models.*

Key-words: cooperative TU-games, non cooperative games, strong Nash equilibrium, multicriteria solutions

1 Introduction

Environmental pollution and the consequent climate change has induced many game theorists to study these topics through games. Consider for example the pollution of shallow lakes, an interesting question studied in *Göran-Maler et al. 2003.* We wish to see crystal water and to have the possibily to admire the submerged plants, fishes and other small animals moving. But often sfortunately, it is not so: there are algae, sediment particles and the water is not clear.
The fishes, searching food, move the sediments so decreasing the trasparency of water.
In the countries where almost all the lakes are shallow, like Denmark or The Netherlands, we can see that they switch back and forth between a clear and a turbid state.

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Many shallow lakes became turbid during the last decay because of eutrophication and the efforts to make them clear have falled. If all decisors cooperate the shallow lakes remain clear, but only a part of players cooperate so they (and we) do not reach the optimum.
In general not all players wish to cooperate to solve a common problem. We consider n players and only some of them cooperate; we speak about semicoperative games or partial cooperative ones. These games arise from models of reality.
This is the ancient problem named "The tragedy of Commons" *Hardin 1968* and deeply studied by many authors among them *Axerold 1988* and *Ostrom 1990.* The overexploitation of commons, the research of selfish own benefit can lead to the worst social output. A cooperative behaviour would be better for all.
Is possible that, among individuals, cooperation arises without a law compelling them? (see *Axelrod 1988*).
Since there is not a istitution which can enforce international environmental agreement (IEAs), there are many difficulties for international cooperation.
To limite the emissions of polluttants, in 1997 Kyoto Protocol to the "Framework Convention on Climate Change" limits emissions of carbon dioxide, methane, nitrous oxide, halocarbons, perfluorocarbons and sulfur hexafluoride, but the solution of the pollution problem is far from a satisfactory solution until now.
Many environmental problems are studied in *Carraro et al. 1993, Murray 1993, Hanley et al. 1997, Fredj et al. 2004*
An interesting approach was made by *Fläm 2006* who studied balanced environmental games that is coalitional games in which the agents decide if cooperate or not and he proves that the cooperating players pollutes less than those who defect. The author proves that the built game has non empty core.
In the last years many game theorists are interested in the problem of coalition formation (see *Aumann et al 1974, Owen 1977, Myerson 1977* and references in these.
Our approach to such problem is to study a cooperative situation modeled by a cooperative game $\langle N, v \rangle$ and to define from it a non-cooperative game which shows the process of coalition formation in the initial cooperative model.
In the applications to real life it is natural to consider multicriteria situ-

ations because a player wishes to "optimize" more criteria. For example a problem is to reduce the pollution and to increase the production, often these two objectives could be opposite.
In the classical approach to TU-games, an allocation rule is defined to predict how rational players distribute among them, the gain from co-operation. In our model we study a game where players have multiple criteria and we propose a new concept of solution (*coalitional Pareto equilibrium*. We use an allocation rule to define the players payoff.
About multicriteria games see *Shapley 1959, Voorneveld 1999, Puerto Albandoz et al. 2006, Patrone et al. 2007, Pieri et al. 2010, Pusillo et al. 2012.*
This paper is so organized: in section 2 we set up notations and known results which will be usefull for our model, in section 3 we describe a non-cooperative model called *claim game*. We give a new idea for the solution of partial cooperative games and an existence result. Section 5 contains conclusions and some ideas for further researches. Many examples illustrate the paper.

2 Notations and Preliminary Results

In this section we consider multiobjective problems.
Let $a, b \in \mathbb{R}^m$ we consider the following preferences:
$a \geqq b \Leftrightarrow a_i \geq b_i \ \forall i = 1, ..., m$;
$a \geq b \Leftrightarrow a \geqq b$ and $a \neq b$;
$a > b \Leftrightarrow a_i > b_i \ \forall i = 1, ..., m$.

Analogously we define $a \leqq b$, $a \leq b$, $a < b$.

Let us consider a multiobjective (or multicriteria) TU- game $\langle N, v \rangle$ where N is the set of n players and $v : 2^N \to \mathbb{R}^m$ is the characteristic function of the game, with $v(\emptyset) = 0$. It assigns to each coalition $S \in 2^N$ a m-vector, being m the number of objectives, equal for each player:

$$v(S) = \begin{pmatrix} v_1(S) \\ v_2(S) \\ \cdots\cdots \\ v_m(S) \end{pmatrix}.$$

Definition 2.1 *A multicriteria game $\langle N, v\rangle$ is convex if it is valid*

$$\alpha_i(S) \leqq \alpha_i(T),$$

$\forall S \subset T$ and $\forall i \in N$, where we mean

$$\alpha_i(S) = \begin{cases} v(S \cup \{i\}) - v(S), & \text{if} \quad i \notin S \\ v(S) - v(S \setminus \{i\}), & \text{if} \quad i \in S. \end{cases}$$

Definition 2.2 *We call i a dummy player if $v(S \cup \{i\}) = v(S) + v(i)$ $\forall S \subset N$.*

Definition 2.3 *The Shapley value of $\langle N, v\rangle$ denoted by $\phi(v)$, is defined, for each player $i \in S$:*

$$\phi_i(v) = \sum_{S \subset N; S \ni i} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S \setminus \{i\})] =$$

$$= \sum_{S \subset N; S \ni i} \frac{(s-1)!(n-s)!}{n!} \begin{pmatrix} v_1(S) - v_1(S \setminus \{i\}) \\ v_2(S) - v_2(S \setminus \{i\}) \\ \dots\dots\dots\dots\dots\dots \\ v_m(S) - v_m(S \setminus \{i\}) \end{pmatrix}.$$

We remind that an *allocation rule* is a map which assigns to every $\langle N, v\rangle$ an element of $\mathbb{R}^n$.
It is not difficult to prove that the Shapley value is an allocation rule verifying the three following axioms.
1) Efficiency property (EFF for short): $\sum_{i \in N} \phi_i(v) = v(N)$

2) Weak Monotonicity property (WMON for short): given two games $\langle N, v\rangle$, $\langle N, w\rangle$ such that

$$\begin{cases} v(S \cup \{i\}) - v(S) \geqq w(S \cup \{i\}) - w(S), & \text{if} \quad i \notin S \\ v(S) - v(S \setminus \{i\}) \geqq w(S) - w(S \setminus \{i\}), & \text{if} \quad i \in S. \end{cases}$$

then $\phi(v) \geqq \phi(w)$.

3) Dummy Out property (DUMOUT for short); for all $\langle N, v\rangle$ and all set of dummies $D \subset N$, it turns out $\phi_i(v) = \phi_i(v_{N\setminus D})$ for all $i \in N \setminus D$ where $v_{N\setminus D}$ is the restriction of the characteristic function v to the set $N \setminus D$ of players.If the dummies of a game abandon it, the others do not dislike.
To explain better the notations let us consider the following:

Example 2.1 Let be $\langle N, v\rangle$ a cooperative game with two criteria:

S	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
$v(S)$	10 15	20 25	30 10	40 50	40 30	60 50	90 80

where we mean $v(\{1\}) = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$, $v(\{2\}) = \begin{pmatrix} 20 \\ 25 \end{pmatrix}$, ...

$v(\{1,3\}) = \begin{pmatrix} 40 \\ 30 \end{pmatrix}$, ... and so on.

The first objective is the reduction of pollution in per cent and the second is the increasing of industrial production. If player 1 decides to make alone, his production increases of 15 per cent but he can reduce his pollution only of 10 per cent. If he cooperates with player 2, their production increases of 50 per cent and they can reduce the 40 per cent of their pollution...,if all players cooperate they increase their production of 80 per cent and they can reduce their pollution of 90 per cent. The Shapley value for this bi-criteria game is:

$$\phi(v) = \begin{pmatrix} 18,3 & 33,3 & 38,3 \\ 22,5 & 37,5 & 20 \end{pmatrix}.$$

□

Let us consider a multicriteria non cooperative game $G = \langle (X_i)_{i\in N}, (u_i)_{i\in N}\rangle$ where $N = \{1, 2, ..., n\}$ is the set of players, X_i is the set of strategies of player i and u_i is the payoff function of player i, $u_i : \prod X_i \to \mathbb{R}^m$, $i = 1, ..., n$.

For $S \subset N$, we write $X_S = \prod_{i \in S} X_i$ and $x_S = x_{|S}$ is the restriction of an $x \in X$ to X_S.

Definition 2.4 *Let G be a multicriteria non-cooperative game.*
A strategy profile $x \in X = \prod X_i$ is a
1) weak Pareto equilibrium if

$$\neg\exists\ y_i \in X_i :\ u_i(y_i, x_{-i}) > u_i(x);$$

2) strong Pareto equilibrium if

$$\neg\exists\ y_i \in X_i :\ u_i(y_i, x_{-i}) \geq\ u_i(x)$$

The set of weak and strong Pareto equilibria will be denoted by wPE and sPE respectively.

Intuitively x^* is a weak (strong) Pareto equilibrium for the game G if there is no strategy y_i which increases the payoff of player i.

Example 2.2 Le G the game in the table below

	L	R
T	$(1,3)$ $(1,2)$	$(2,1)$ $(2,2)$
B	$(1,5)$ $(0,0)$	$(0,2)$ $(3,3)$

In pure strategies the equilibria are:
$wPE = \{(T,L),(T,R),(B,R)\}$
$sPE = \{(T,R),(B,R)\}$. □
We recall the definition of *strong Nash equilibrium* (sNE for short) for a scalar non cooperative game as studied in *Ichiishi 1983*, *Borm et al. 1992*, *Marini et al. 2011.*

It is well known that a NE is not necessarily a Pareto optimum. The hypotheses about sNE are to supply conditions under which there exist Pareto optimal points.

Definition 2.5 *Given a non-cooperative scalar game G, we say that $x \in X$ is a strong Nash equilibrium of G (for short sNE) if* $\neg S \subset N$, $S \neq \emptyset$ *and* $y_S \in X_S$ *such that* $u_j(y_S, x_{N\setminus S}) \geq u_j(x), \quad \forall j \in S$ *and* $u_k(y_S, x_{N\setminus S}) > u_k(x) \quad$ *for some* $k \in S$.

Intuitively there is no coalition S such that, if all players of S deviate they do not gain more. This is a NE which is a Pareto optimal too. In the following example 2.3 (emission reduction game), we show the non existence of a sNE, in the example 2.4 (emission reduction game generalized) we show the existence of a sNE. See *Ichiishi 1983*.

Example 2.3 We consider a symmetric game, so it is sufficient to write only the payoff of the first player.

	C	nC
C	50	-40
nC	60	-30

Here two countries I, II, have to decide as to reduce sulphur dioxide emissions. Each country has two strategies: to cut (C) its emissions or not to cut (nC). Because sulphur dioxide is a pollutant, each country is affected by the emissions of the other one.
The payoffs depend from the control cost to avoid damages. The game is represented in the matrix, negative numbers are the losses.
If player I does not cut but player II decides to cut, then the first has no control costs and he has a benefit from the reduction of emissions of the second, so his payoff is +60, but player II has control cost and he has no benefit from second country reduction. So II will have -40 as payoff.
The efficient solution is (C,C) but as in the Prisonner Dilemma the NE is (nC,nC) and it is not a sNE.
There are no sNE in this game. □

Let us consider now an emission game generalized, it is not a symmetric game. The payoffs are in the table below.

Example 2.4

	C	nC
C	$(70, 50)$	$(-20, 40)$
nC	$(60, -20)$	$(-30, -30)$

The Nash equilibrium is (C,C) and it is a sNE too. In this case the equilibrium lead to the social fitness. $\square$

3 From a coalition game to a strategic one

In many practical situations, the interaction among decisors may be a mixture of cooperative and non-cooperative behavior.
The players in the same coalition cooperate, there are also coalitions formed by a unique player because he does not wish to cooperate. Furthermore coalitions do not act cooperatively.
In such models we speak of *partial cooperative games*, see *Mallozzi e al. 2008, Ayoshim et al. 2000.*
Let us consider $\langle N, v\rangle$ a multicriteria cooperative TU-game and an allocation rule ϕ which is agreed by all players.
By $\langle S, v_s\rangle$ we mean the restriction of the TU-game $\langle N, v\rangle$ to the coalition S. By $\phi(v_s)$ we mean the restriction of the allocation rule of the game $\langle N, v\rangle$ to the game $\langle S, v_s\rangle$.

We can define the non cooperative *claim game* in normal form
$G = \langle (X_i)_{i\in N}, (\pi_i)_{i\in N}\rangle$ in the following way: $X_i = \{x_i \in 2^N : i \in x_i\}$ that is the strategies are the coalitions to which the player i would like to belong.

The utility function for player i is defined as follows:

$$\pi_i(x) = \pi_i(x_1, x_2, ..., x_n) = \begin{cases} \phi_i(v_{x_i}); & if \quad x_j = x_i \quad \forall j \in x_i, \\ v(\{i\}), & \text{in any other case.} \end{cases}$$

Where $\pi_i(x) = \left(\begin{array}{c} \pi_i^1(x) \\ \pi_i^2(x) \\ \ldots\ldots \\ \pi_i^m(x) \end{array} \right)$

that is π_i is a vector of m components and the same for $\phi_i(v_{x_i})$ and $v(\{i\})$.

Intuitively we start with a TU-game and an allocation rule agreed by all the players. Then we trasform it into a strategic game where the strategies of players are the coalitions to which they wish to belong (they propose the coalitions simultaneously and independently from the other players). A coalition is formed if all its members have proposed it. The utility function gives to each player the value of the allocation rule if the coalition forms and the value of the coalition formed by the single player ($v(\{i\})$) in any other case.

As first example let us consider a one criterium cooperative game where the allocation rule is the Shapley value.

Example 3.1 Let us consider the TU-game $\langle\{1, 2, 3, 4\}, v\rangle$ defined in the following table:

S	$v(S)$	$\phi_1(v_S)$	$\phi_2(v_S)$	$\phi_3(v_S)$	$\phi_4(v_S)$
$\{1\}$	1	1	–	–	–
$\{2\}$	1	–	1	–	–
$\{3\}$	0	–	–	0	–
$\{4\}$	3	–	–	–	3
$\{1,2\}$	5	2.50	2.50	–	–
$\{1,3\}$	0	0.50	–	−0.50	–
$\{1,4\}$	3	0.50	–	–	2.50
$\{2,3\}$	0	–	0.50	−0.50	–
$\{2,4\}$	3	–	0.50	–	2.50
$\{3,4\}$	0	–	–	−1.50	1.50
$\{1,2,3\}$	6	3	3	0	–
$\{1,2,4\}$	0	0	0	–	0
$\{1,3,4\}$	0	0.66	–	−2.33	1.66
$\{2,3,4\}$	0	–	0.33	−1.66	1.33
$\{1,2,3,4\}$	0	0.83	0.83	−0.83	−0.83

table 3.3.1

The first column of this table contains the coalitions and the second one the value of the corresponding coalition. In the others $\phi_i(v_S)$ is the Shapley value for player i if the coalition S forms and if we consider the restricted game with players in S. For example if $S = \{2,4\}$, in the second column there is $v(\{2,4\}) = 3$ and there are, in the following columns, $\phi_2(v_{\{2,4\}}) = 0.50$; $\phi_4(v_{\{2,4\}}) = 2.50$. Obviously ϕ_1 and ϕ_3 do not appear.

Remark that the initial cooperative game has empty core but the corresponding claim game has NE. There are many NE but a unique sNE: $y = (\{1,2,3\},\{1,2,3\},\{1,2,3\}\{4\})$ □

In the following definition we generalize the concept of strong NE to a notion of equilibrium profile for multicriteria games. We call it *coalitional Pareto equilibrium.*

Definition 3.1 *We say that $x \in X$ is a coalitional Pareto equilibrium of G (for short cPE) if*
$\neg\exists S \subset N$, $S \neq \emptyset$ *and* $y_S \in X_S$ *such that* $\pi_i(y_S, x_{N\setminus S},) \geqq \pi_j(x) \quad \forall j \in S$
$\pi_k(y_S, x_{N\setminus S},) \geq \pi_k(x) \quad$ *for some* $k \in S$.

Intuitively x is a cPE if there is no coalition S such that if all the players in S deviate, they do not make worse for each objectives and at least one player gains strictly on at least one objectif. We can note that if x^* is a cPE with an allocation rule, it could be not a cPE with another allocation rule. If the players have only one objective to optimize the cPE is a sNE.

Definition 3.2 *We say that a multicriteria partial cooperative game has a partial equilibrium if there is a cPE for the corresponding claim game G .*

Example 3.2 Let us consider the following bicriteria game.

S	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
$v(S)$	0	0	0	2	2	2	0
	1	1	0	5	0	0	6

;

Let us call ϕ the allocation rule. We can use the following table to see quickly the coalitional Pareto equilibria:

S	$v(S)$	$\phi_1(v_S)$	$\phi_2(v_S)$	$\phi_3(v_S)$
$\{1\}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	–	–
$\{2\}$	$\begin{pmatrix}0\\1\end{pmatrix}$	–	$\begin{pmatrix}0\\1\end{pmatrix}$	–
$\{3\}$	$\begin{pmatrix}0\\0\end{pmatrix}$	–	–	$\begin{pmatrix}0\\0\end{pmatrix}$
$\{1,2\}$	$\begin{pmatrix}2\\5\end{pmatrix}$	$\begin{pmatrix}1\\5/2\end{pmatrix}$	$\begin{pmatrix}1\\5/2\end{pmatrix}$	–
$\{1,3\}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}1\\1/2\end{pmatrix}$	–	$\begin{pmatrix}1\\-1/2\end{pmatrix}$
$\{2,3\}$	$\begin{pmatrix}2\\0\end{pmatrix}$	–	$\begin{pmatrix}1\\1/2\end{pmatrix}$	$\begin{pmatrix}1\\-1/2\end{pmatrix}$
$\{1,2,3\}$	$\begin{pmatrix}0\\6\end{pmatrix}$	$\begin{pmatrix}0\\3\end{pmatrix}$	$\begin{pmatrix}0\\3\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$

table 3.2.1

There are two cPE :
$(\{1,2\},\{1,2\},\{3\})$

and $(\{1,2,3\},\{1,2,3\},\{1,2,3\})$

If we decompose this bicriteria game in two games (the first one relative to the first criterium, the second one relative to the second criterium), we can remark that $(\{1,2\},\{1,2\},\{3\})$ is not a sNE for any considered game, $(\{1,2,3\},\{1,2,3\},\{1,2,3\})$ is a sNE only for the second one.

We remark that table 3.2.1 permit us to arrive speedy to the conclusion.
In general we have to study the strategic game:
$G = (X_1, X_2, X_3, \pi_1, \pi_2, \pi_3)$ where the set of players is $N = \{1,2,3\}$,
$X_1 = \{\{1\},\{1,2\},\{1,3\},\{1,2,3\}\}$;
$X_2 = \{\{2\},\{1,2\},\{2,3\},\{1,2,3\}\}$; $X_3 = \{\{\{3\},\{1,3\},\{2,3\},\{1,2,3\}\}$.
The utility functions are defined consequently.
We remark that some strategies profile are equivalent, for example $(\{1\},\{2,3\},\{3\})$; ({1,2,3},{1,2},{3}); ({1},{1,2,3},{3}) lead to the same profile $(\{1\},\{2\},\{3\})$. Studying the cPE we arrive to the same conclusions as in table 3.2.1. □

4 Existence theorem

Lemma 4.1 *Let v be a scalar TU-cooperative convex game. Let ϕ an allocation rule with the WMON and DOMOUT properties then $\forall S \neq \emptyset$, $S \subset N$ it turns out $\phi_i(v) \geq \phi_i(v_S)$, $\forall i \in S$.*

Proof: see *Meca et al. 1998*.
We have the following existence theorem:

Theorem 4.1 *Let v be a multicriteria convex TU game and ϕ let an allocation rule with the WMON and DOMOUT properties then $\{N, N, ..., N\}$ is a cPE for the multicriteria game.*

Proof
Let us suppose, by contradiction, that $x = \{N, N, ..., N\}$ is not a cPE;

then $\exists S \subset N, S \neq \emptyset, y_s \in X_s$ s.t.
$\pi_i(y_S, x_{N \setminus S}) \geqq \pi_i(x) \quad \forall i \in S$
$\pi_k(y_S, x_{N \setminus S}) \geq \pi_k(x) \quad \text{for some } k \in S$

In other words there is at least a component $\bar{l} \in \{1, ..., m\}$ such that
$\phi_i^{\bar{l}}(v_S) \geq \phi_i^{\bar{l}}(v) \ \forall i \in S$
and
$\exists j \in S : \phi_j^{\bar{l}}(v_S) > \phi_j^{\bar{l}}(v)$.
This is a contradiction with the lemma 4.1. □

5 Conclusion and Open problems

Starting from a game where some players cooperate and others do not, we give a new concept of equilibrium for multicriteria partial cooperative game, the *coalitional Pareto equilibrium* which generalizes the idea of strong Nash. An existence theorem and many examples complete the paper.
About possible further research it would be interesting to study the partial cooperative problem where the allocation rule is the Alexia value (see *Tijs (2005)*) or to consider games with utility functions in uncertainty intervals, (see *Pieri et al. 2010*).
Some of these issues are work in progress.

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