A Game Theoretic Approach to Emergency Units Location Problem

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Abstract

The location of the emergency units may have a deep influence on the intervention time, which in its turn may be relevant for a positive solution of an emergency situation. In this paper we analyze a particular location problem with a discontinuous intervention time function. The problem is tackled from different points of view each one resulting in a different game. The paper refers to a toy model and to a real world situation of locating some additional ambulance in the province of Milan.

Key Words

Emergency Management, Cooperative Game, Location Problem

1 Introduction

Emergency management represents a hard task in several situations. It has to be viewed under many different lights, so that a large number of parameters is involved. Consequently, many different experiences and expertizes are needed in order to have an efficient result. First of all, it is necessary to distinguish among different types of emergency: medical, environmental, urban, and so on. Each of them has particular features that have to be tackled in different ways, with different means and tools. Before going deeper in our analysis we want to devote

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some words to make clear the characteristics of the kind of situations that are more suitable for our approach.

The idea of emergency is strongly correlated to the idea of urgency, and then with the idea of fast intervention. On the other hand the measure of the time required is different for the various types of emergency. Medical intervention should be carried out in minutes, while environmental emergency may allow longer time and the rescue missions after an earthquake can be organized also some days after the event.

In this work we are interested in those particular situations for which the utility of the intervention has a discontinuity, i.e. after a given period there is a sudden and large reduction of the necessity of the intervention. A classical example is represented by the ambulances that are allowed for a maximal time for the most difficult situations, the so-called *red codes*; for example in Italy the time allowed is 8 minutes, including the time for answering the call. It is clear that if an ambulance arrives after 9 minutes the intervention is anyhow completed (for example an injured person is transported to the hospital), but when an ambulance is located on the territory only the area that can be reached in 8 minutes is considered as covered, when a statistical analysis is performed. Another interesting situation is represented by fire planes or helicopters, for which it is necessary to take into account the fuel tank capacity and the distance from a place for charging water. Summarizing, in the standard location problem the utility of a service located in a fixed point decrease when the distance from the point increases, while in this work we refer to situations in which the utility can be consider maximal until a given distance and then it can be considered equal to zero. We want to remark that the distance may be measured in different ways, e.g. ground distance, time distance, linear distance.

The main idea that has driven us in this problem was to consider the different candidate locations as agents interacting among them. In fact the choice of deploying an emergency unit in a candidate location cannot take into account only its own characteristics, i.e. the extension of the area that can be covered in the maximal allowed time, the probability of a call in that area, but should consider also the location of the other emergency units. This starting point enables us to have a game theoretic approach to the problem, in order to emphasize the interactions among the ambulances deployed on the area under analysis. Location problems where already studied with the instruments of Game Theory, see [2], where cost sharing problems where studied, but our situation is the first case in which the game theoretic approach is used to support the localization of the facilities.

Finally, we want to stress that the complexity of the problem of emergency management allows and requires the synergic use of different methods and approaches. Referring to various aspects of the problem, we can mention, besides Game Theory, statistical analysis, simulation, topography and GIS (Geographical Information System) , graph theory, mathematical programming, and stochastic optimization.

The present paper is organized as follows: in the next section we recall some definitions and solutions of cooperative game theory; in Section 3 we introduce two TU games, each of them focusing on particular features of the problem, and analyze a simple way for aggregating the results of them; Section 4 is devoted to a simple theoretical model for testing our approach; a more complex situation arising from the real world is presented in Section 5; Section 6 concludes.

2 Preliminaries on Game Theory

In this section we introduce some basic game theoretical concepts that we use throughout the paper. A *cooperative game with transferable utility* or TU-game, is a pair (N, v), where $N = \{1, 2, ..., n\}$ denotes the finite set of *players* and $v : 2^N \rightarrow I\!R$ is the *characteristic function*, with $v(\emptyset) = 0$. Often, a TU-game (N, v) is identified with the corresponding characteristic function v. A group of players $S \subseteq N$ is called a *coalition* and $v(S)$ is the *worth* of the coalition.

Given a TU-game (N, v), an *allocation* is a vector $(x_i)_{i\in N} \in I\!R^n$ assigning to player $i \in N$ the amount x_i. An allocation $(x_i)_{i\in N}$ is *efficient* if $\sum_{i\in N} x_i = v(N)$. A *solution* for a TU-games is a function ψ that assigns an allocation $\psi(v)$ to every TU-game in the class of games with player set N. Game theoretic literature proposes several solutions for TU-games; in this paper we consider the *Shapley value* and the *Banzhaf value*, introduced by Shapley [3] and by Banzhaf [1], respectively.

The Shapley value assigns to each player his average marginal contribution over all the possible permutations of the players. Formally, given a game (N, v), the Shapley value assigns to player $i \in N$:

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi} (v(P(\pi; i) \cup \{i\}) - v(P(\pi; i))$$

where π is a permutation of the players and $P(\pi; i)$ is the set of players that precede player i in the permutation π.

The Banzhaf value assigns to each player his average marginal contribution over all the possible coalitions he does not belong to. Formally, given a game (N, v), the Banzhaf value assigns to player $i \in N$:

$$\beta_i(v) = \frac{1}{2^{n-1}} \sum_{S \in N \setminus \{i\}} (v(S \cup \{i\}) - v(S))$$

The Shapley value is an efficient allocation, while the Banzhaf value is not. In general, it is necessary to normalize the Banzhaf value in order to reach efficiency, when it is used to allocate a profit, a cost or simply to estimate the value of a player in a voting situation. Here, we look for ranking the candidate locations for the emergency units, so that no normalization is necessary.

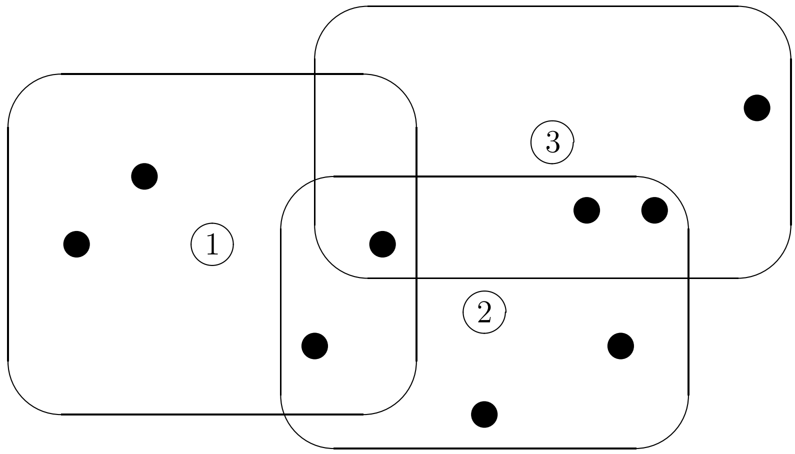
In the following section we will introduce two TU games, each focusing on different features of the problems, and will use the Shapley value and the Banzhaf value to estimate the relevance of each candidate location.

3 The Games

The first idea for tackling the problem is to take into account the number of emergency calls that each subset of candidate locations may satisfy, In order to simplify we model we suppose that all the calls from the area covered by an ambulance may be satisfied by the ambulance itself, i.e. the time between two calls is larger than the time required for completing the first intervention.

We may define a TU game (N, v^1), where the player set N corresponds to the set of candidate locations and the characteristic function v^1 associates to each coalition $S \subseteq N$ the worth $v^1(S)$ that is given by the number of calls covered locating the ambulances only in the locations represented by S. Consider the following example.

Example 1 *Consider an area with 9 calls, represented by the bullets; there exist 3 candidate locations, 1, 2 and 3, represented by a circle, and each of them may cover the area represented by the rectangle centered in it.*



In this case it is possible to associate the game

$$v^1(1) = 4; v^1(2) = 6; v^1(3) = 4; v^1(12) = 8; v^1(13) = 7; v^1(23) = 7; v^1(123) = 9$$

In order to determine the relevance of each candidate location we decided to use the Shapley value and the Banzhaf value. The motivation is that these solutions are rooted in the concept of marginal contribution and this is a very good way for representing the interaction of the "players" because, as we said in the Introduction the relevance of a location depends on the number of satisfied call that may be added, on the average.

For the situation in Example 1 we obtain:

$$\phi = (2.833, 3.833, 2.333)$$

$$\beta = (2.750, 3.750, 2.250)$$

and in both cases the ordering is 2, 1, 3.

We had the doubt that the results of this game could be affected when two different locations overlap for a large number of calls, so that both may have a great relevance even if only one of them is actually important. For better analyzing these situations we defined a second TU game (N, v^2), with the same player set N and where the characteristic function v^2 associates to each coalition $S \subseteq N$ the worth $v^2(S)$ that is given by the number of calls covered by more than one ambulance in S.

For the situation in Example 1 we obtain:

$$v^2(1) = 0; v^2(2) = 0; v^2(3) = 0; v^2(12) = 2; v^2(13) = 1; v^2(23) = 3; v^2(123) = 4$$

For solving this game we propose again the Shapley value and the Banzhaf value, for the same motivation. In this case the highest the value, the highest the average multiple coverage of calls, so that the ordering has to be reverted. For this game we obtain:

$$\phi = (0.833, 1.833, 1.333)$$

$$\beta = (1.000, 2.000, 1.500)$$

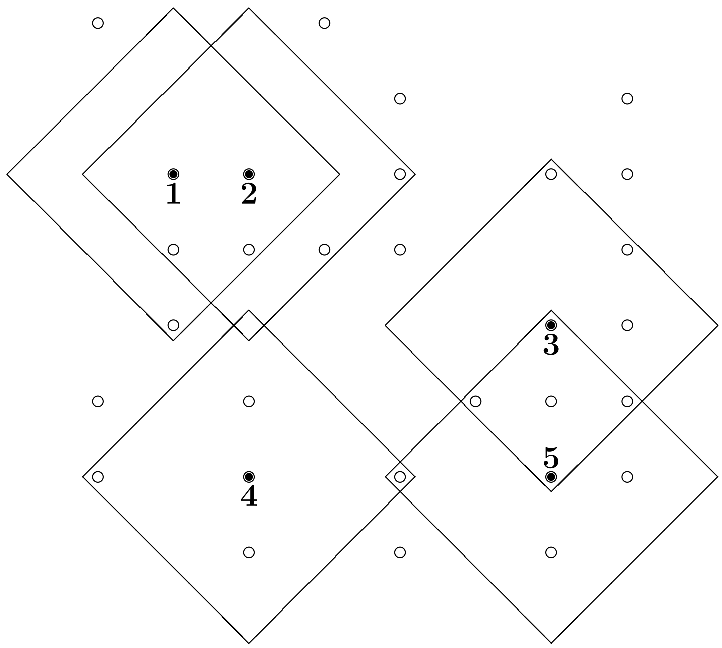
and in both cases the ordering is 1, 3, 2.

Also for this game, it is necessary to remark that an isolated location, i.e. whose covered area has empty intersection with the area of each other location, has always a null marginal contribution; so, its Shapley value and Banzhaf value is always zero and this location has always the highest ranking. For this reason, we decide to take into account both orderings via a Borda count. For each ordering, we assigned 1 point to the first location, 2 to the second one and so on until the last that obtains n points; summing up the points of the locations we ordered them for increasing values.

Referring again to Example 1, we have: 3 points (2 + 1) for the location 1, 4 points (1 + 3) for the location 2 and 5 points (3 + 2) for the location 3; so, the ranking according to the Borda count is 1, 2, 3.

4 A Case Study

In this section we present the characteristics of the games proposed in the previous section applying them to a simple example. Suppose that an area is described as a set of 30 zones distributed on a unitary grid, each corresponding to a circle in the following figure. We can consider that all the zones have the same uniform distribution of the probability of generating an emergency call. In addition, suppose that there exist 5 zones that are candidate locations for an ambulance, represented by a bullet inside the corresponding circle, and it is possible to deploy 3 ambulances. We suppose that the intervention time is such that an ambulance may move two steps on the grid, and the distance is measured according to the Manhattan metric, i.e. moves are allowed only along the vertical and the horizontal directions [3]. This metric is very suitable for representing distances in an urban area, where it is necessary to follow the streets that usually cross orthogonally.



Note that it will be necessary to have further candidate locations and ambulances in order to cover all the zones, or it will be necessary to increase the maximal travel time of the ambulances, so that they may reach zones at a distance of four units. For a complete analysis of the situation in each row of the following tables we report the results supposing that ambulances may move from one to four steps. We consider both the game 1, i.e. taking into account the number of covered zones (second column), and the game 2, i.e. penalizing the multiple coverage of the zones (third column); the last column represents the mix of the

[3] Mathematically, the Manhattan metric corresponds to the metric associated to the unitary norm.

two previous results when the Borda count is applied. Measuring the candidate locations with the Shapley value, we get the following results:

# of steps	v^1	v^2	*Borda count*
1	$5, 4, 3, 1 - 2$	$4, 3 - 5, 1 - 2$	$4 - 5, 3, 1 - 2$
2	$3, 5, 4, 2, 1$	$4, 1 - 2, 3 - 5$	$4, 3, 2 - 5, 1$
3	$3, 2 - 4, 5, 1$	$4, 1, 2, 3, 5$	$4, 2 - 3, 1, 5$
4	$3, 4, 5, 2, 1$	$4, 1 - 2, 5, 3$	$4, 2 - 3, 1 - 5$

The sign "$-$" among two locations, instead of the comma, means that they have the same value and, consequently there is a draw in the ordering.

Measuring the candidate locations with the Banzhaf value, we get the following results:

# of steps	v^1	v^2	*Borda count*
1	$5, 4, 3, 1 - 2$	$4, 3 - 5, 1 - 2$	$4 - 5, 3, 1 - 2$
2	$3, 5, 4, 2, 1$	$4, 1 - 2, 3 - 5$	$4, 3, 2 - 5, 1$
3	$3, 2 - 4, 5, 1$	$4, 1, 2 - 3, 5$	$4, 3, 2, 1, 5$
4	$3, 4, 5, 1 - 2$	$4, 1, 2, 5, 3$	$4, 3 - 1, 2 - 5$

It is easy to notice that there are small differences among the orderings arising from the Shapley value and the Banzhaf value; also the difference due to the different number of steps are not very strong, except between one step and the others; on the opposite the two games produce larger differences, justifying our idea of introducing both and the use of the Borda count in order to balance the characteristics of the two approaches.

Finally, we want to remark that the same example was used by two researchers of the University of Milan that approached the location ambulance problem using a mathematical programming method, with results consistent with ours. On the other hand our approach may have a dynamic interpretation. Given an ordering of the locations, we have the solution also when the number of ambulances varies; suppose that we select the case in which ambulances move two steps and then we decide to refer to the Borda count ordering for the Shapley value approach, this corresponds to deploy the three ambulances in locations 4, 3, 2 (or 5). If a further ambulance becomes available it will be located at 5 (or 2); on the opposite, if one of the three ambulances is temporarily out of order, it is possible to decide to use locations 4 and 3.

5 A Real-world Example

After the encouraging results obtained testing our game theoretic models on the simple situation presented in the previous section, we had the possibility to apply them to a real case, which has some features that match very well those of our models. The 118 of Milan, a public institution that manages the ambulances in the area of the province, had to locate 5 further ambulances. Before going on we shortly introduce the main features of this situation.

Referring to 2005, 118 received 576,562 emergency calls, i.e. an average of 1579.62 calls per day, 247,594 of which required the intervention of an ambulance. In the area under consideration there are 27 ambulances already operating. The time for answering an emergency call is averaged at 2 minutes, including the time for defining the urgency of intervention (red, yellow or green code) and for identifying the place where the intervention is needed; as the maximal allowed intervention time is 8 minutes, the area covered by an ambulances corresponds to the area that can be reached in 6 minutes. An exact evaluation of the area is a difficult task that depends on various parameters: the road network, the density of traffic in the different periods of the day, the instant of the call. The candidates locations are the hospitals and the seats of emergency instititions, for a total of about 80. In order to simplify this aspect we made the following hypotheses and approximations:

- the new ambulances may be located only at the hospitals; in this way we reduced the number of players from about 80 to 10;
- the ambulances have a constant speed; it depends only on the density of inhabitants in the municipality where the hospital (candidate ambulance location) is located;
- the distance is measured according to the Euclidean norm; so, the area covered by an ambulance is a circle;
- the ray of each circle depends only on the speed of the ambulance, i.e. on the density of inhabitants of the location of the hospital; we suppose that

the speed, v, vary among 20 and 70 Km/h according the following formula:

$$v = \frac{50}{\rho^{0.2}} + 20$$

where $\rho \in [1, +\infty[$ is the normalized density of population, with 1 corresponding to the density of the municipality of Morimondo (with the minimum of 45.910 inhabitants per Km^2), and for the other municipalities it is obtained dividing the real density by 45.910.

- the emergency calls are uniformly distributed w.r.t. the number of inhabitants.

We want to remark that in the real situations the number of players is usually very high. For example in the urban area of Milan it is possible to locate an ambulance quite in every place, because it is sufficient to provide a space for parking the ambulance waiting for a call and a phone in order to alert the crew for an intervention.

We considered the following 10 hospitals:

	Hospital	Municipality
1	P.O. Cuggiono	Cuggiono
2	P.O. Magenta	Magenta
3	P.O. Bollate	Bollate
4	Ospedale di Garbagnate	Garbagnate Milanese
5	Humanitas	Rozzano
6	P.O. Cernusco S.N.	Cernusco sul Naviglio
7	Multimedica	Sesto San Giovanni
8	P.O. Sesto San Giovanni	Sesto San Giovanni
9	P.O. Bassini	Cinisello Balsamo
10	Ospedale di Saronno	Saronno

whose characteristics are summarized in the following table.

	Ab	A	d	C	CAb	CK	ρ	v	r	cA
1	7,816	14.780	528.820	329	0.0421	22.260	11.519	50.668	5.067	80.613
2	23,330	21.820	1,069.200	1,285	0.0551	58.891	23.289	46.640	4.664	68.305
3	37,556	15.910	2,360.530	1,911	0.0509	120.113	51.416	42.738	4.274	57.353
4	27,167	8.870	3,062.800	1,425	0.0525	160.654	66.713	41.584	4.158	54.297
5	38,493	12.230	3,147.420	2,841	0.0738	232.298	68.556	41.466	4.147	53.991
6	28,687	13.470	2,129.700	1,560	0.0544	115.813	46.389	43.211	4.321	58.629
7	83,415	11.740	7,105.200	6,148	0.0737	523.680	154.764	38.240	3.824	45.917
8	83,415	11.740	7,105.200	6,148	0.0737	523.680	154.764	38.240	3.824	45.917
9	73,935	12.710	5,817.070	4,641	0.0628	365.146	126.706	38.985	3.899	47.723
10	37,472	10.840	3,456.830	2,426	0.0647	223.801	75.296	41.068	4.107	52.958

where:	Ab	inhabitants
	A	municipal area (in Km^2)
	d	inhabitants per Km^2 (density)
	C	calls per day
	CAb	calls per inhabitant
	CK	calls per Km^2
	ρ	normalized density
	v	average ambulance speed (in Km/h)
	r	ray of the area covered in 6 minutes
	cA	covered area (in Km^2)

Figure 1 represents the situation we faced; the dotted circles correspond to the area covered by the existing 27 ambulances and the full circles represent the area that an ambulance located in a candidate hospital may cover.

The results are summarized in the following table:

	v^1	v^2	*Borda count*
Shapley	$3, 1, 7, 4, 8, 5, 2, 10, 9, 6$	$1, 2, 3, 4, 6, 9, 8, 7, 10, 5$	$1, 3, 4, 2, 7, 8, 6-9, 5, 10$
Banzhaf	$3, 1, 7, 4, 8, 5, 2, 10, 9, 6$	$1, 2, 3, 9, 6, 4, 8, 7, 10, 5$	$1, 3, 2, 4, 7, 8, 9, 6, 10, 5$

6 Concluding Remarks and Further Researches

The results obtained up to now are satisfactory and encouraging for future researches, nevertheless, some ground remains. The hypothesis of not interfering calls is a strong one; on the other hand our main target is to collect information

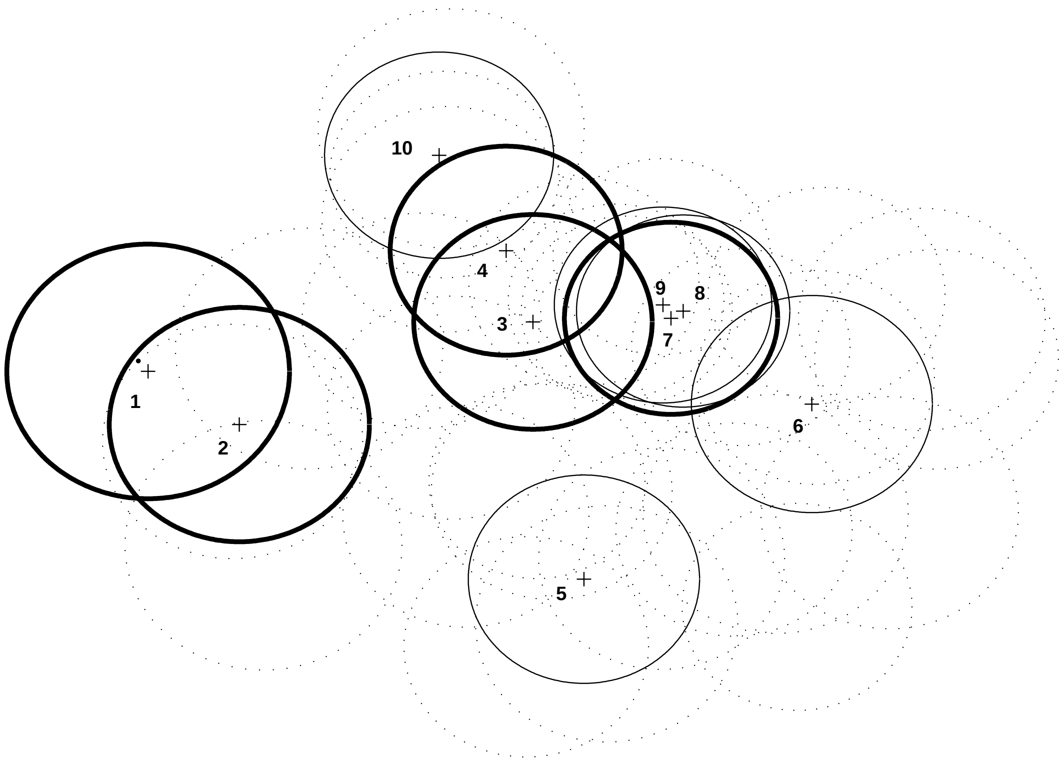


Figure 1: Hospital locations and covered areas

on the possibility of getting a good deployment of the ambulances using Game Theory, more than evaluating the quality of the service and this result can be considered as successfully reached. Of course it is possible to improve the model supposing that an emergency unit may serve only a limited number of calls per day, and consequently to consider the possibility of having more than one player associated to each location, especially in those areas with a high frequency of calls. Another refinement of the game is to assign different values to the areas covered by more than one location, as multiple coverage corresponds to a better service. On the other hand we may note that, when a large emergency occurs, for example a large road accident, the situation is managed coordinating different emergency units for example firemen and civil defence.

An important aspect of the problem is the computational complexity that influences both the definition of the games and the solutions. As the players correspond to the candidate location of the emergency units, it is possible that their number may be very high; we can mention as a good situation the location of fire planes that can be located only in the airports; another well suitable situation is the real case presented in Section 5, in which the candidate locations for the ambulances corresponds to the hospitals. On the opposite, the location of ambulances in urban areas where no constraint is fixed may give rise to thousands of candidate locations.

A possible development is the following. Referring to the case study in Section 4, suppose that ambulances from locations 4 and 2 are carrying out an intervention each. If the one located at 2 is the first to finish the intervention it could be redirected to location 4 that is more relevant; but we can do also better, for example we may decide to move the ambulance from location 3 to location 4. Of course, this possibility requires a deep analysis using simulative and statistical methods.

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